Many-Body Vortex Effect on the Transverse Force Acting on a Moving Vortex

Nobuhiko Hayashi
Computer Center, Okayama University, Okayama 700-8530, Japan
(August 23, 1999)

The nondissipative transverse force acting on one moving vortex under the influence of another vortex is discussed in fermionic superfluid systems, where the relative velocity between the vortices is finite. On the basis of detailed numerical solutions of the Bogoliubov-de Gennes equation, the Berry phase for an adiabatic motion of the vortex line is examined for a two-vortex system. It is found that the detailed electronic structure of a vortex core can affect the transverse force, without abandoning the previous discussions of the robust Magnus force on a single vortex.

PACS number(s): 47.32.Cc, 74.60.Ec, 74.60.Ge

Much interest has been devoted to the dynamical motion and pinning effect of vortices existing in various situations in nature such as type-II superconductors[1], superfluidity in ^{3}He, and superfluid neutron star matters.[2,3] It is essential also from the applied physics point of view that we understand transport properties due to vortices in type-II superconductors, especially for high-T_c cuprates, to better control superconducting characteristics under magnetic fields. In addition, the recent progress of experiments on ^{3}He in a rotating cylinder shed new light on the fundamental physics field where exciting analogies between the early Universe and the superfluid ^{3}He were examined.[4,5] For these systems, the dynamics of moving vortices is the focal point.

In this letter, we consider a *many-body effect* between vortices on the dynamics of a vortex. We will propose that a novel force should act on one moving vortex due to surrounding another vortex when the relative velocity between the vortices is finite.

Currently, in the case of fermionic superfluids and superconductors, the fundamental equation of motion for a moving vortex is highly controversial. In particular, the term of the nondissipative transverse force in the equation of motion, i.e., the vortex velocity dependent part of the Magnus force (per unit length of the vortex axis z), $\alpha\hat{z}\times\mathbf{v}_L$, is under intensive debate.[6–9] Here, $\mathbf{v}_L$ is the velocity of a vortex.

Broadly speaking, in the single vortex case, at present there are two conflicting points of view on the theory of the transverse force. One theory[10–13] considers a momentum transfer from the (vortex) system under consideration to a heat bath system by a relaxation of the quasiparticles of the vortex bound states[14] inside a vortex core. In this theory, called the spectral flow theory,[13] the coefficient of the transverse force, α, essentially depends on the vortex bound states (i.e., the electronic structure inside a vortex core) in combination with the relaxation time τ of the quasiparticles. Experimental results on ^{3}He in a rotating cylinder support the spectral flow theory,[15] which is the crucial point of the ^{3}He simulation of the early Universe.[4] In contrast, the other theory[16,17] claims that the transverse force on a moving vortex is a robust quantity which never depends on such quasiparticle relaxation inside a vortex core. This "robust" Magnus force does not depend on the details of the vortex bound states inside a vortex core, but only on the superfluid density far from the core[17] as long as a single vortex is considered. There also exist experimental results supporting the robust Magnus force theory,[18] and a negative experimental result for the spectral flow theory is obtained there. Thus, the transverse force issue is controversial now, and careful interpretation of the experimental results by generalizing the single-vortex theories mentioned above to more complicated situations will be required to resolve it, as noted in ref. 9. To advance our knowledge of vortex dynamics, it is important to consider *many-body effects* between vortices whose relative velocity is nonzero, due to, for example, vortex pinning.

The transverse force is also a key point in the understanding of the Hall effect in the mixed state in type-II superconductors,[19] especially for the mysterious sign reversal of the Hall resistivity (conductivity) observed experimentally in several type-II superconductors.[20] A typical objection to the robust Magnus force theory has arisen from small Hall angles observed in most transport experiments, because the robust (or full large) Magnus force leads to an extremely large Hall angle and to no sign reversal, as long as the many-body effect is not considered. However, in real superconductors, the vortex pinning cannot be avoided, and then it is expected that the relative velocity of vortices is generally nonzero, leading to many-body effects between vortices. Ao[21] previously proposed that the movement of vortex vacancies in the pinned vortex lattice as a many-body vortex effect should lead to a relevant result and the sign reversal, while he maintained the robust Magnus force acting on individual moving vortices. Recent transport experiments[22] certainly revealed that the vortex pinning played an important role in the Hall effect in some way. Therefore, it is required to further investigate what type of *many-body effect* can exist in various possible vortex situations, (e.g., in the plastic flow state, in which portions of the vortex lattice move while other portions remain pinned[23]), i.e., situations in which the relative velocity of vortices is generally nonzero.

In addition, in high-T_c cuprate superconductors, it was

universally observed that the sign-reversal Hall effect existed only in underdoped samples, but not in overdoped ones.[24] A hypothetical concept arises from this result; the change of the electronic state due to the doping could be related to the internal electronic structure inside vortex cores so that it affects the dynamic property of vortices. Such an idea opens a new possibility of relating the doping-dependent Hall effect[24] to the electronic structure of various vortex cores currently the focus of attention, e.g., cores with the antiferromagnetic moment,[25] in $d_{x^2-y^2} + id_{xy}$ superconductivity,[26] and with doping dependence.[27]

Motivated by the above concepts, we investigate a contribution from the electronic structure around a vortex core to the transverse force acting on a moving vortex *under the influence of another vortex* (e.g., pinned one). We consider such a *many-body effect* between vortices whose relative velocity is nonzero, by discussing a two-vortex system. A Berry phase picked up by the system for an adiabatic vortex motion is examined to derive the coefficient of the transverse force $\alpha\hat{z} \times \mathbf{v}_L$, according to the Berry phase approach to the vortex dynamics,[16,28,29] on the basis of numerical wave functions around each vortex. It should be noted that the relaxation time τ is not included in the present analysis, and therefore the contribution of the vortex core which we propose is independent of the spectral flow theory[13] which essentially depends on τ. We will consider a neutral system and rectilinear vortices (or vortices in a two-dimensional system) with vorticity antiparallel to $\hat{z}$.

We will base our analysis on the Bogoliubov-de Gennes (BdG) theory,[30] which is the spatially inhomogeneous version of the BCS theory. The system is described in terms of the Bogoliubov wave function $\big(u_j(\mathbf{r}),\ v_j(\mathbf{r})\big)$. We start with the BdG equation given, in a dimensionless form, by

$$\begin{pmatrix} K & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -K \end{pmatrix} \hat{\chi}_j(\mathbf{r}) = E_j \hat{\chi}_j(\mathbf{r}), \quad \hat{\chi}_j(\mathbf{r}) = \begin{pmatrix} u_j(\mathbf{r}) \\ v_j(\mathbf{r}) \end{pmatrix}, \quad (1)$$

where $K = -\nabla^2/2k_{\rm F}\xi_0 - \mu$, μ is the chemical potential, and $\xi_0 (= v_{\rm F}/\Delta_0)$ is the coherence length [Δ_0 is the uniform gap at zero temperature $T = 0$, $k_{\rm F}$ ($v_{\rm F}$) is the Fermi wave number (velocity), and $\hbar = 1$]. In eq. (1), the length (energy) is implicitly measured by ξ_0 (Δ_0). The system is characterized by a parameter $k_{\rm F}\xi_0$.[31] The pair potential is self-consistently determined with $\Delta(\mathbf{r}) = g\sum_{|E_j|\le\omega_{\rm D}} u_j(\mathbf{r})v_j^*(\mathbf{r})\{1 - 2f(E_j)\}$. Here, g is the coupling constant and $\omega_{\rm D}$ the energy cutoff, which are related by the BCS relation via the transition temperature $T_{\rm c}$ and the gap Δ_0. We set $\omega_{\rm D} = 20\Delta_0$.

To obtain exact solutions $\hat{\chi}_j(\mathbf{r})$ (i.e., electronic structure) around a vortex, we numerically calculate eq. (1) under the following conditions for clarity, as in ref. 31. (a) The system is a cylinder with radius R. (b) The Fermi surface is cylindrical. (c) The pairing has isotropic s-wave symmetry. Thus, the system has cylindrical symmetry. We write the eigenfunctions as $u_j(\mathbf{r}) = u_{n,l}(r)\exp\big[i(l - \frac{1}{2})\theta\big]$ and $v_j(\mathbf{r}) = v_{n,l}(r)\exp\big[i(l + \frac{1}{2})\theta\big]$ with $\Delta(\mathbf{r}) = \Delta(r)\exp\big[-i\theta\big]$ in cylindrical coordinates, where n is the radial quantum number and the angular momentum $|l| = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots$. We expand the eigenfunctions in terms of the Bessel functions [14] $J_m(r)$ as [32] $u_{n,l}(r) = \sum_i c_{ni}\phi_{i|l-\frac{1}{2}|}(r)$, $v_{n,l}(r) = \sum_i d_{ni}\phi_{i|l+\frac{1}{2}|}(r)$, where $\phi_{im}(r) = [\sqrt{2}/RJ_{m+1}(\alpha_{im})]J_m(\alpha_{im}r/R)$, α_{im} is the i-th zero of $J_m(r)$, and $i = 1, 2, \cdots, N$. We set $R = 20$–$40\ \xi_0$. Equation (1) is reduced to a $2N \times 2N$ matrix eigenvalue problem. This useful technique to solve eq. (1), developed by Gygi and Schlüter,[32] has been utilized in many cases.[2,26,31,33]

Adiabatic vortex motion produces a Berry phase ϕ_j in the solution of eq. (1), $\hat{\chi}_j \to \exp[i\phi_j]\hat{\chi}_j$, and the total Berry phase Γ picked up by the whole system, $|\Phi\rangle \to \exp[i\Gamma]|\Phi\rangle$, is given as $\Gamma = -\sum_j \phi_j$ [28,29].

We consider the single vortex case first. When the center of the vortex $\mathbf{r}_0(t)$ adiabatically moves, the motion gives rise to the Berry phase ϕ_j picked up by the Bogoliubov wave function $\hat{\chi}_j$. With the cylindrical symmetry of the vortex, the total Berry phase is obtained as[28,29]

$$\begin{aligned} \Gamma &= -\int dt \int d^2x(\dot{\mathbf{r}}_0\nabla_{\mathbf{r}_0}\theta)S(r) \\ &= -\pi\big[S(\infty) - S(0)\big]\int dt(x_0\dot{y}_0 - y_0\dot{x}_0), \end{aligned} \quad (2)$$

where $S(r)$ is the canonical angular momentum density composed of $\hat{\chi}_j$, and $\dot{\mathbf{r}}_0 = \mathbf{v}_L$. In the framework of the BdG theory, the physical quantity $S(r)$ must be expressed *in the form with the Fermi distribution function* $f(E)$, as pointed out by Gaitan,[28] namely

$$\begin{aligned} S(r) = 2\sum_{E_{n,l}>0} \Big[&-f(E_{n,l})|u_{n,l}(r)|^2\Big(-l + \frac{1}{2}\Big) \\ &+ \{1 - f(E_{n,l})\}|v_{n,l}(r)|^2\Big(-l - \frac{1}{2}\Big)\Big]. \end{aligned} \quad (3)$$

At $T = 0$, eq. (3) which is calculated using our numerical solutions (u_j, v_j) of eq. (1) gives $S(0) = 0$ and the uniform value $S = -\rho/2$ far from the vortex core (see Fig. 1), where $\rho/2$ is half the total particle density and is, at $T = 0$, equal to the density of the Cooper pair, i.e., half the superfluid density $\rho_s/2$. Thus, the expression for the Magnus force derived at $T = 0$ by Ao and Thouless,[16] $\alpha = -h\rho_s/2$, is obtained from eqs. (2) and (3) as in refs. 28 and 29. In the present study, we extend the above discussion for $T = 0$ to the finite temperature case within the mean-field BdG framework and with an ansatz that, at finite temperatures, the total Berry phase is the sum of the thermally weighted contribution from each Bogoliubov wave function, i.e., the total Berry phase should be obtained from eq. (2) by calculating the finite-temperature canonical angular momentum density $S(r)$ composed of u_j, v_j, and $f(E_j)$ (eq. (3)). It is consistent with an imaginary time path integral formulation.[34] We

should note here that the solutions of the BdG equation eq. (1), i.e., the Bogoliubov wave functions (u_j, v_j), have *all the information on the system* on both the condensate and non-condensate in a two-fluid picture.

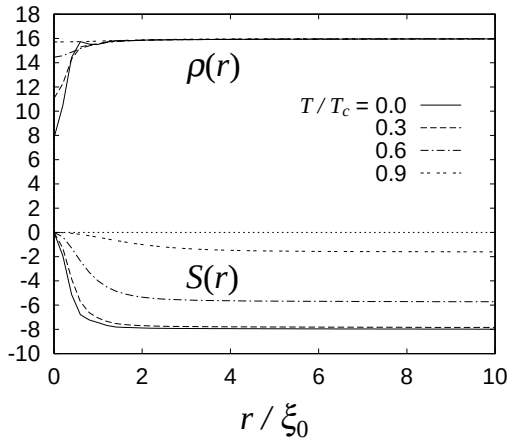

FIG. 1. The canonical angular momentum density $S(r)$ and the total particle density $\rho(r)$ around a vortex at several temperatures, in arbitrary units, but on a common scale. $k_F\xi_0 = 4$. Note the relation $S = -\rho/2$ at $T = 0$ far from the vortex center $r = 0$.

In Fig. 1, we show $S(r)$ at several temperatures. $S(0)$ remains zero at finite temperatures in Fig. 1 due to the form of eq. (3) and the necessary structure of $u_{n,l}(r)$ and $v_{n,l}(r)$ expanded by the Bessel functions. Because $S(0) = 0$, even at finite temperatures the transverse force does not depend on the vortex core in the single vortex case. The particle density,[33] $\rho(r) = 2\sum_{E_j>0}\big[|u_j(r)|^2 f(E_j) + |v_j(r)|^2\{1 - f(E_j)\}\big]$, is simultaneously presented in Fig. 1. Here, we adjust the chemical potential μ at each temperature so that the particle density ρ far from the vortex core is invariable.[33] With increasing temperature, $|S|$ decreases with respect to $\rho/2$. We can find from Fig. 1 that the decrease of $|S|$ far from the core obeys the temperature dependence of the Yosida function.[35] This result gives numerical proof of the physical estimation in ref. 34. Then, $|S|$ indicates the superfluid density $\rho_s/2$ at finite temperatures.

Let us consider the two-vortex problem, which is the main aim in the present study. We consider two vortices, labeled 1 and 2, moving with finite relative velocity. Their relative dynamics is of interest here. Hence, without loss of generality, vortex 1 is at rest at position $\mathbf{r}_1$ and vortex 2 at $\mathbf{r}_2(t)$ moves around vortex 1. We assume the vortices are separated so that there is almost no overlap between the vortex bound states of each vortex. Then, wave functions bounded inside each vortex core are defined separately from each other. A two-vortex pair potential is expressed in the product form, $\Delta(\mathbf{r}, t) = \Delta_v(\mathbf{r} - \mathbf{r}_1)\Delta_v(\mathbf{r} - \mathbf{r}_2(t))$, where $\Delta_v(\mathbf{r}) = \Delta(r)\exp[-i\theta]$ denotes the pair potential of the single vortex. The wave functions near each vortex are expected to be close to those of the single vortex which have cylindrical symmetry, but a phase factor is attached so that the above product form of the pair potential is constructed from those wave functions.

Under the above conditions, we obtain the bound-state contribution to the Berry phase[29,36] as $\Gamma_b = \Gamma_{1b} + \Gamma_{2b}$; the contribution from the bound-state wave functions of vortex 1 is $\Gamma_{1b} = -2\pi A \int dt(R_x\dot{R}_y - R_y\dot{R}_x)/|\mathbf{R}|^2$, and that of vortex 2 is $\Gamma_{2b} = -2\pi A \int dt(R_x\dot{R}_y - R_y\dot{R}_x)/|\mathbf{R}|^2 - \pi S_b \int dt(x_2\dot{y}_2 - y_2\dot{x}_2)$, where $\mathbf{R} = \mathbf{r}_2(t) - \mathbf{r}_1$, $A = \int rdrP(r)$, (the center of each vortex is $r = 0$), and

$$P(r) = -2 \sum_{0<E_{n,l}<\Delta(T)} \frac{1}{2}\Big[|u_{n,l}(r)|^2 f(E_{n,l}) + |v_{n,l}(r)|^2\{1 - f(E_{n,l})\}\Big]. \quad (4)$$

The second term in Γ_{2b} is the bound-state contribution to the Berry phase which leads to the Magnus force in the absence of vortex 1, and is zero ($S_b = 0$) as seen in the single vortex case. What we aim to show in this letter is that the bound-state contribution $P(r)$ is actually nonzero. In the previous two-vortex discussion,[29] this "$|u_j|^2 - |v_j|^2$ type" contribution of the bound states has been disregarded because of the equality $|u_j| = |v_j|$ for approximated bound-state solutions. However, exact solutions in general must have essential asymmetry between $|u_j|$ and $|v_j|$ due to the existence of the vortex itself, which leads to a local breaking of particle-hole symmetry in the density of states inside the vortex core[31] and an electric charging of the core.[33] The Fermi functions should also be attached to the Bogoliubov wave functions on the analogy of $S(r)$ in eq. (3). In Fig. 2, we show $P(r)$ which is calculated with the exact solutions (u_j, v_j) around the vortex, and find it is certainly nonzero.

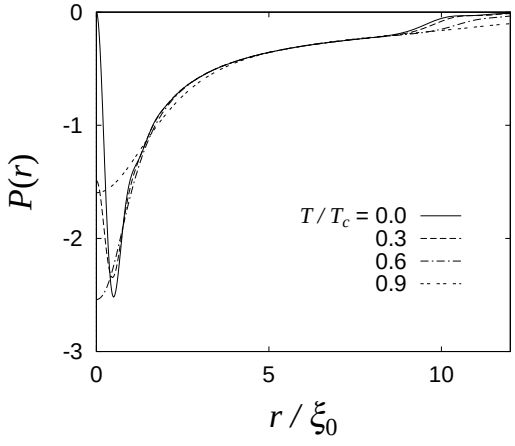

FIG. 2. Plot of $P(r)$ (in arbitrary units) obtained using exact wave functions of the vortex bound states around a vortex (see text). $k_F\xi_0 = 4$.

On the other hand, the wave functions of the extended

states above the gap $\Delta(T)$ spread over vortices 1 and 2. To consider their contribution, let us apply a discussion by Dziarmaga[29] based on an analogy of the impurity effect on the Magnus force due to Ao and Thouless.[16] The Berry phase is proportional to the mean number of superfluid particles enclosed by a closed path around which a vortex is moved adiabatically.[37] The stationary vortex 1, as a kind of impurity, traps particles and excludes them from the extended states relevant to the Berry phases picked up by the extended wave functions. A quantity which becomes a central issue is $\delta S_e(r) = S_e(r) + \rho_s/2$,[29] the deviation of S_e from its asymptotic value $-\rho_s/2$, where $S_e(r)$ is the contribution from the extended states to $S(r)$ in eq. (3) around the stationary vortex 1. Then $2\int dr 2\pi r \delta S_e(r)$ $(\equiv B)$ corresponds to the excluded particle number which works to decrease the magnitude of the single-vortex Magnus force $|-h\rho_s/2|$ in spatial average.[29] The contribution to the Berry phase is $\delta\Gamma = -B\int dt(R_x\dot{R}_y - R_y\dot{R}_x)/|\mathbf{R}|^2$.

Taking the variation of the Berry phase $\Gamma_b + \delta\Gamma$ about $\mathbf{r}_2$, we finally obtain the force $\mathbf{F}_2$ acting on vortex 2 due to the presence of vortex 1, $\mathbf{F}_2 = \big(h(4A+B/\pi)/|\mathbf{R}|^2\big)\big[\hat{z}\times\dot{\mathbf{R}} + \hat{z}\cdot(\mathbf{R}\times\dot{\mathbf{R}})\mathbf{R}/|\mathbf{R}|^2\big]$, and we propose to add it to the general form[21] of the usual vortex equation of motion. In addition to the nondissipative transverse force (the first term in large brackets), we obtain an interesting dissipative term (the second one) which, to our knowledge, has not been reported so far. An analysis of this dissipative term is a future task. In what follows, we discuss the nondissipative transverse force.

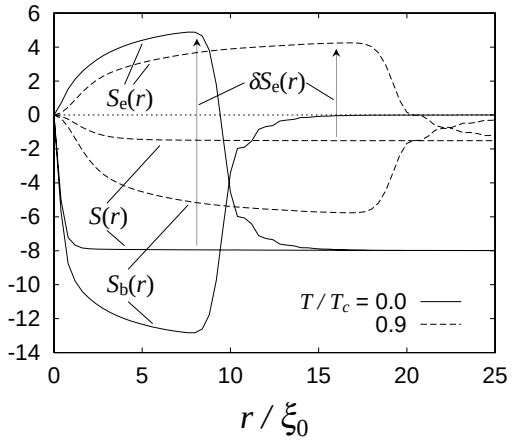

FIG. 3. To the canonical angular momentum density $S(r)$ around a vortex (in arbitrary units), the contribution from the vortex bound states $S_b(r)$ and that from the extended states $S_e(r)$, where $S(r) = S_b(r) + S_e(r)$. The deviation $\delta S_e(r) = S_e(r) + \rho_s/2$ (see text). $k_F\xi_0 = 4$.

In Fig. 3, we show our numerical results for $S_e(r)$ together with the bound-state contribution $S_b(r)$ and total $S(r)$. The deviation $\delta S_e(r)$ is positive as noted in ref. 29. It is of importance to note here that the deviation can become $\delta S_e(r) > |-\rho_s/2|\big(=|S(\infty)|\big)$ locally around the vortex core, as seen in Fig. 3. We are able to understand this by the fact[32] that in general the extended states contribute in terms of negative rotation to the supercurrent around a vortex, and the bound states contribute in terms of positive rotation to it. We note in Fig. 3 that at a higher temperature, the region of such large $\delta S_e(r)$ expands following the divergence of the coherence length near T_c. Accordingly, near T_c the region around such stationary vortices where $\delta S_e(r) > |-\rho_s/2|$, can predominantly occupy the inside of, e.g., a superconductor, and thus the transverse force acting on the moving vortex can become not only a decreased Magnus force but also an opposite force. It may naturally explain the sign-reversal Hall effect near T_c observed experimentally.[20,24] The detailed structure of $\delta S_e(r)$ can depend on the details of various vortex cores, and thus on the doping[27] in high-T_c cuprates.

Effects of surrounding vortices on a vortex are usually taken into account through the superfluid velocity dependent part of the Magnus force, $\beta\hat{z}\times\mathbf{v}_s$, where $\mathbf{v}_s$ is the superfluid velocity including the contribution of the surrounding vortices. In the present study, in contrast, we proposed quite a different force due to the surrounding vortices, $\alpha\hat{z}\times\mathbf{v}_L$, which acts on a moving vortex. According to Newton's action-reaction principle, our transverse forces acting on the moving vortex simultaneously act on the surrounding vortices in each opposite direction. If the surrounding vortices are pinned ones, our force should be included in a depinning condition for those vortices.

In conclusion, we considered how the details of the vortex core can affect the vortex dynamics by the many-body vortex effect other than the possibility of the spectral flow force which is due to τ. The present study throws a light on the possibility for the transverse force which may depend on the details of vortex cores, without abandoning the robust Magnus force theory for the single vortex.

A numerical simulation adopting the robust Magnus force and the force $\mathbf{F}_2$, and a direct two-vortex system analysis by numerically calculating the BdG for a system containing two vortices, would be interesting and are left for future work.

I am grateful to Ping Ao for bringing my attention to his work and for helpful discussions.

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