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Theoretikum zur Einführung in die Theoretische Festkörperphysik  
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### Exercise Set 7

(Due date: Tuesday, December 8, 2009)

#### Exercise 16 (Baker-Campbell-Hausdorff formula) (20 points)

Consider two operators  $\hat{A}$  and  $\hat{B}$  with  $[\hat{A}, [\hat{A}, \hat{B}]] = 0$  and  $[\hat{B}, [\hat{A}, \hat{B}]] = 0$ .

Prove the simple form of the Baker-Campbell-Hausdorff formula

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-1/2[\hat{A}, \hat{B}]}.$$

**Hint:** Consider the function

$$f(x) = e^{\hat{A}x} e^{\hat{B}x},$$

and show that  $f(x)$  obeys the differential equation

$$\frac{df}{dx} = (\hat{A} + \hat{B} + [\hat{A}, \hat{B}]x)f(x).$$