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Exercise Set 6

(Due date: Tuesday, December 1, 2009)

Exercise 13 (**Debye model**) (10 points)

Consider a two-dimensional square lattice with one atom of mass m per lattice point interacting with only nearest neighbors with force constant K. The phonon dispersion curve is given by

$$\omega_q = \sqrt{\frac{4K}{m}} \sin\left(\frac{qa}{2}\right)$$

a) In the long-wavelength limit, obtain the phonon density of states $D(\omega) = dN/d\omega$.
Hint: On the 2D lattice, you can find the Debye wave number q_D by considering a Debye circle instead of a Debye sphere.

b) For high temperature ($k_B T \gg \hbar\omega$), calculate the internal energy.

Exercise 14 (**Specific heat**) (10 points)

The density of modes of a monoatomic crystalline solid is given by

$$D(\omega) = \frac{3V}{2\pi^2 c_0^3}\omega^2 ,$$

where c_0 is the speed of sound in the crystal and V is the volume. What is the specific heat of the monoatomic crystalline solid at high temperature?

Exercise 15 (**Quantum corrections**) (10 points)

Determine the most important quantum corrections to the Dulong-Petit law by calculating the high temperature expansions of the internal energy and the specific heat up to terms proportional to $1/T$ and $1/T^2$, respectively.