Okayama University
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Okayama, January 21, 2020

Exercises for Advanced Physics 2, 2019/20 term 4

Exercise Set 5

(Due date: Tuesday, January 28, 2020)

Exercise 5 (**Coulomb interaction in second quantization**) (10 points)

(a) Read Chapter 5.1 of the script and explain the idea of second quantization in your own words.

(b) When rewriting the Coulomb interaction using second quantization, we encounter the term

(1) $$\sum_{\sigma_1\sigma_2} a^\dagger_{\vec{R}m_1\sigma_1} a_{\vec{R}m_1\sigma_2} a^\dagger_{\vec{R}m_2\sigma_2} a_{\vec{R}m_2\sigma_1}$$

Use the anticommutation relations of the Fermionic creation (annihilation) operators $a^\dagger_{\vec{R}m_1\sigma_1}$ ($a_{\vec{R}m_1\sigma_1}$) and the definitions

(2)
$$n_{\vec{R}m} = \sum_\sigma a^\dagger_{\vec{R}m\sigma} a_{\vec{R}m\sigma}$$
$$s^\alpha_{\vec{R}m} = \sum_{\sigma\sigma'} a^\dagger_{\vec{R}m\sigma} \frac{\sigma^\alpha_{\sigma\sigma'}}{2} a_{\vec{R}m\sigma'} \quad \text{(Pauli matrices } \sigma^\alpha_{\sigma\sigma'}, \alpha = x, y, z\text{)}$$
$$\vec{s}_{\vec{R}m_2} = \left(s^x_{\vec{R}m}, s^y_{\vec{R}m}, s^z_{\vec{R}m}\right)$$

of the number operators $n_{\vec{R}m}$ and the spin operators $s^\alpha_{\vec{R}m}$ to show that

(3) $$\sum_{\sigma_1\sigma_2} a^\dagger_{\vec{R}m_1\sigma_1} a_{\vec{R}m_1\sigma_2} a^\dagger_{\vec{R}m_2\sigma_2} a_{\vec{R}m_2\sigma_1} = \frac{1}{2} n_{\vec{R}m_1} n_{\vec{R}m_2} + 2\vec{s}_{\vec{R}m_1} \cdot \vec{s}_{\vec{R}m_2}$$