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Exercises for Advanced Physics 3, 2018 term 4

Exercise Set 3

(Due date: Tuesday, January 8, 2019)

Exercise 3 (**Classical Langevin paramagnetism**) (10 points)

Consider a classical thermodynamic system with N atoms at sites i in a volume V. Each atom has a constant magnetic moment μ_i with $|\mu_i| = \mu$ for all i. The Hamiltonian function contains a part $H_0(\vec{q}, \vec{p})$ describing the system in the absence of a magnetic field, and a part $H_1(\vec{q}, \vec{p})$ discribing the influence of the homogeneous magnetic field $\vec{B}_0 = B_0 \vec{e}_z$:

$$H(\vec{q}, \vec{p}) = H_0(\vec{q}, \vec{p}) + H_1(\vec{q}, \vec{p})$$

(a) Explain why H_1 should read

$$H_1 = -\mu B_0 \sum_{i=1}^{N} \cos\theta_i$$

if θ_i is the angle between the magnetic moment μ_i and the magnetic field.

(b) Calculate the canonical partition function.

(c) Determine the dependences on temperature and magnetic field of the average total magnetic moment

$$\vec{m} = \Big\langle \sum_{i=1}^{N} \mu_i \Big\rangle$$

(d) Discuss the total magnetic moment in the limits $\beta\mu B_0 \gg 1$ and $\beta\mu B_0 \ll 1$ where $\beta = \frac{1}{k_B T}$.