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Exercises for Advanced Physics 3, 2018 term 4

Exercise Set 1
(Due date: Tuesday, December 18, 2018)

Exercise 1 (**Landau diamagnetism**) (10 points)

Consider the two-dimensional electron gas in the presence of a perpendicular field $\vec{B}_0 = B_0\vec{e}_z$. In the plane, assume that the electron gas is enclosed in a rectangular sample with side lengths L_x and L_y. According to section 3.3.1 of the script, in the ground state, the N electrons occupy the Landau levels

$$E_{n,k_x} = \hbar\omega_c\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots \qquad (1)$$
$$\omega_c = \frac{eB_0}{m}$$

The spin splitting is neglected here.

(a) What is the smallest field $B_0 = B_0^{(0)}$ at which all the electrons are placed in the $n = 0$ level?

(b) What is the field $B_0 = B_0^{(n_0)} \leqslant B_0^{(0)}$ at which the N electrons are uniformly distributed in the Landau levels up to the quantum number n_0?

(c) If the field B_0 is between the two critical fields $B_0^{(n_0)}$ and $B_0^{(n_0-1)}$

$$B_0^{(n_0-1)} \geqslant B_0 \geqslant B_0^{(n_0)},$$

calculate the total energy $E(B_0)$ of the N electron system.

(d) What is the result for the special case $E\big(B_0^{(n_0)}\big)$?

Please explain all steps!