How does neutrino confine GUT and Cosmology?

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T. Fukuyama (Rits) @ Center of Quantum Universe, Okayama–U
1. Introduction

- Neutrino oscillation breaks SM.
- Then is $\text{SM} + \nu_R$ OK?
- $\text{SM} + \nu_R$ does not predict

1. Gauge couplings unification
2. Hierarch problem of Higgs mass
3. DM and DE problem

→ SUSY GUT
Overview of experimental data

• **Neutrino masses & mixings**
  - Since the discovery of neutrino oscillations in SuperK (1998), we have a first evidence of physics beyond the SM.
  - Now is in the era of precision measurement for atmospheric & solar neutrino oscillations in SuperK, K2K, SNO and KamLand.
  
  \[ \Delta m_{23}^2 = 2.1 \times 10^{-3} eV^2, \quad \Delta m_{12}^2 = 8.3 \times 10^{-5} eV^2 \]
  \[ \sin^2 \theta_{23} = 0.5, \quad \sin^2 \theta_{12} = 0.28, \quad \sin^2 \theta_{13} \leq 0.061 \]

  \[ \nu_{2004} \]

  - Future prospect is to measure the remaining mixing angle, CHOOZ angle, and \( \theta_{13} \) P violation in the lepton sector!
  - These data show that we have two large mixing angles plus one small angle in the lepton sector. (cf. quark sector have three small mixing angles.)
There exist a 2.8σ discrepancy between the recent BNL result and the Standard Model predictions.

If this is real, it may be a first evidence of supersymmetry.
Motivation of SO(10) GUT

• **SUSY + GUT**
  - Experimental evidence: three gauge couplings unification with MSSM particle contents.

• **SO(10) GUT**
  - SO(10) fundamental representation include all the matter in the MSSM plus right-handed neutrinos.
  - Experimental evidence: very tiny neutrino masses.
    - It can be explained via the "seesaw mechanism", and it works well in SO(10) GUT in the presence of the right-handed neutrinos. (Yanagida, Gell-Mann et al. (79'))
Minimal SO(10) model

(Babu-Mohapatra (93’); Fukuyama-Okada (01’))

• Two kinds of symmetric Yukawa couplings

\[ W_Y = Y_{10}^{ij} 16_i H_{10} 16_j + Y_{126}^{ij} 16_i H_{126} 16_j \]

• Two Higgs fields are decomposed to

\[ 10 \rightarrow (6, 1, 1) + (1, 2, 2) \]
\[ 126 \rightarrow (6, 1, 1) + (15, 2, 2) + (\overline{10}, 3, 1) + (10, 1, 3) \]

• SU(4) adjoint 15 have a basis, \( \text{diag}(1, 1, 1, -3) \) so as to satisfy the traceless condition. Putting leptons into the 4 \(^{th} \) color, we get, so called, ‘Georgi–Jarlskog’ factor, \(-3\) leptons.
Perturbative (to the Planck scale) SUSY SO(10)

• Raby, Albright, Babu–Barr, ⋯

\[
W_Y = 16_i 16_j (Y^{ij}_{10} 10_H \\
+ Y^{ij}_{45} 10_H 45_H / M + Y^{ij}_{16} 16_H 16_H / M + \ldots)
\]
Why renoramalizable?

• Renoramalizability is the guiding principle for the SM. Remember anomalous free condition.
• For SU(3)xU(1)_em Fermi interaction is the effective interaction.
• For SM model it becomes renoramalizable interaction but neutrino mass is the effective mass.
• At GUT nu_R becomes massless and it is described by the renoramalizable Yukawa coupling.
Yukawa couplings

- After the symmetry breakings, we have

\[ W_Y = (u_R^c)_i \left( Y_{10}^{ij} H_{10}^u + Y_{126}^{ij} H_{126}^u \right) q_j \]
\[ + (d_R^c)_i \left( Y_{10}^{ij} H_{10}^d + Y_{126}^{ij} H_{126}^d \right) q_j \]
\[ + (\nu_R^c)_i \left( Y_{10}^{ij} H_{10}^u - 3Y_{126}^{ij} H_{126}^u \right) \ell_j \]
\[ + (e_R^c)_i \left( Y_{10}^{ij} H_{10}^d - 3Y_{126}^{ij} H_{126}^d \right) \ell_j \]
\[ + (\nu_R^c)_i \left( Y_{126}^{ij} v_R \right) (\nu_R^c)_j \]

Below the GUT scale, we assume MSSM is realized, and we have two Higgs doublet which are linear combinations of original fields.
Predictions in neutrino sector

- We have only one parameter \( \sigma = \arg(c_d) \) left free. So, we can make definite predictions.

For \( \sigma = 3.198 [\text{rad}] \):
\[
\sin^2 2\theta_{12} \sim 0.72, \quad \sin^2 2\theta_{23} \sim 0.90, \quad \sin^2 2\theta_{13} \sim 0.16
\]
Yukawa’s are determined!

- Now, all the mass matrices have been determined!
- For example, Neutrino Dirac Yukawa coupling matrix (in the basis where charged lepton mass matrix is diagonal):

\[
Y_\nu = 
\begin{pmatrix}
-0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\
0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\
-0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i
\end{pmatrix}
\]

- We must check this model by proving the other phenomena related to the Yukawa couplings! (LFV, muon g–2, EDM, neutrino magnetic dipole moment (MDM), proton decay, etc.)
We consider the general soft SUSY breaking mass parameters at low-energy.

\[ -\mathcal{L}_{\text{soft}} = \bar{\ell}_i^\dagger \left( m_\ell^2 \right)_{ij} \bar{\ell}_j + \bar{\nu}_{Ri}^\dagger \left( m_\nu^2 \right)_{ij} \bar{\nu}_{Rj} + \bar{e}_{Ri}^\dagger \left( m_e^2 \right)_{ij} \bar{e}_{Rj} \\
+ m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d \\
+ \left( B \mu H_d^\dagger H_u + \frac{1}{2} B \nu M_{Rij} \bar{\nu}_{Ri}^\dagger \bar{\nu}_{Rj} + h.c. \right) \\
+ \left( A^i_{\nu} \bar{\nu}_{Ri}^\dagger \bar{\ell}_j H_u + A^i_{e} \bar{e}_{Ri}^\dagger \bar{\ell}_j H_d + h.c. \right) \\
+ \left( \frac{1}{2} M_1 \tilde{B} \tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{G}^a \tilde{G}^a + h.c. \right) \]
Lepton Flavor Violation (LFV)

- Because we really see LFV as neutrino oscillations, we naturally expect LFV can also be seen in the charged lepton sector!
- Current experimental bound:
  \[
  \text{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}
  \]
  \[
  \text{BR}(\tau \rightarrow \mu\gamma) < 0.6 \times 10^{-6}
  \]
- **How well motivated from theoretical point of view?**
  - In the Standard Model (+ Right–handed neutrinos): too small rate (∵ GIM suppression well works).
  - In SUSY models: New source of LFV, soft SUSY breaking terms (with No GIM suppression, in general) exist.

**LFV processes are important sources for low–energy SUSY search!**
Soft SUSY breaking terms

- We consider the general soft SUSY breaking mass parameters at low-energy

\[ -\mathcal{L}_{\text{soft}} = \ell_i^\dagger (m_\ell^2)_{ij} \tilde{\ell}_j + \tilde{\nu}_{Ri}^\dagger (m_{\tilde{\nu}}^2)_{ij} \tilde{\nu}_{Rj} + \tilde{e}_{Ri}^\dagger (m_{\tilde{e}}^2)_{ij} \tilde{e}_{Rj} \]

\[ + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d \]

\[ + \left( B\mu H_d^\dagger H_u + \frac{1}{2} B_{\nu} M_{R_{ij}} \tilde{\nu}_{Ri}^\dagger \tilde{\nu}_{Rj} + \text{h.c.} \right) \]

\[ + \left( A_{\nu}^{ij} \tilde{\nu}_{Ri}^\dagger \tilde{\ell}_j H_u + A_{\tilde{e}}^{ij} \tilde{e}_{Ri}^\dagger \tilde{\ell}_j H_d + \text{h.c.} \right) \]

\[ + \left( \frac{1}{2} M_1 \tilde{B} \tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{G}^a \tilde{G}^a + \text{h.c.} \right) \]
mSUGRA boundary condition

- We impose the universal boundary conditions at the GUT scale

\[
(m^2_{\ell})_{ij} = (m^2_{\nu})_{ij} = (m^2_{\tilde{e}})_{ij} = m^2_0 \delta_{ij},
\]

\[
m^2_{H_u} = m^2_{H_d} = m^2_0,
\]

\[
A^{ij}_\nu = A^{ij}_\tau, \quad A^{ij}_{\tilde{e}} = A^{ij}_{\tilde{\nu}}
\]

\[
M_1 = M_2 = M_3 = M_{1/2}
\]

- So we have three free parameters in the soft mass terms,

\[
m_0, \ M_{1/2} \ \text{and} \ A_0.
\]
RG induced Flavor mixings

• Flavor mixing is induced by RG running from the GUT scale to the EW scale

\[ \frac{d}{d\mu} (m^2_{\ell})_{ij} = \frac{d}{d\mu} (m^2_{\ell})_{ij} \mid_{\text{MSSM}} + \frac{1}{16\pi^2} \left( m^2_{\ell} Y^\dagger Y + 2 Y^\dagger m^2_{Y} Y + Y^\dagger m^2_{W} Y + 2 Y^\dagger m^2_{H} Y \right)_{ij} \]

• In the basis where the charged lepton mass matrix is diagonal, LFV is induced through the neutrino Dirac Yukawa coupling matrix.

\[ \left( \Delta m^2_{\ell} \right)_{ij} \sim -\frac{3m^2_0 + A^2_0}{8\pi^2} \left( Y^\dagger L Y \right)_{ij} \]

where \( L_{ij} = \log \left( \frac{M_{R_i}}{M_G} \right) \delta_{ij} \)
• **Proton Decay Results**: The decay rate in our model is within the experimental upper bound if we take the small \( \tan \beta' = 2.5 \), assuming the color triplet Higgs mass to be at the usual GUT scale.

\[
\tau(p \rightarrow K^+ \bar{\nu}) \leq 2 \times 10^{33} \left( \frac{10}{\tan \beta} \right)^2 \left( \frac{m_S}{10 \text{ [TeV]} \right)^2 \\
\times \left( \frac{M_C}{2 \times 10^{16} \text{ [GeV]} \right)^2 \text{ [years]}.
\]

• Note added: It’s possible to completely cancel out the proton decay rate by tuning the parameters in the Higgs sector (3 +1 or 5+1 parameters).

**Our SO(10) Model is very testable in the near future experiments!**
Superpotential was fully analyzed

- Fukuyama et.al. hep-ph/0401213 v1 gave the symmetry breaking pattern from minimal SO(10) to Standard Model, starting from

\[ W = m_1 \Phi^2 + m_2 \Delta \overline{\Delta} + m_3 H^2 + \lambda_1 \Phi^3 + \lambda_2 \Phi \Delta \overline{\Delta} + \lambda_3 \Phi \Delta H + \lambda_4 \Phi \overline{\Delta} H. \]

\[ H = 10, \Delta = 126, \Phi = 210 \]
Many vacua (STD singlets)

- (1234)
- (5678+5690+7890) 210
- (1256+1278+1290+3456+3478+3490)
- (13579) 126
- (24680) 126
2. Problems of 4D SO(10) GUT

- Thus the theory enter into the precision calculation phase, where some problems give arise:
- Fast Proton Decay.
- Many intermediate energy scales break the gauge coupling unification since we have five directions which are singlet under $G_{321}$. 
The gauge coupling unification

From Bertolini-Scwetz-Malinsky hep-ph/0605006
Solution by warped extra dimension
–First approach–

1. A variety of Higgs mass spectra destroys the successful gauge coupling unification in the MSSM. Especially, the existence of the intermediate mass scale for the right-handed neutrino cause a problem.

\[ M_R \ll M_G \iff \text{gauge coupling unification} \]

2. This model has a cut off scale at the GUT scale. It means that a concrete UV completion of the model is necessary to be considered.

\[ \alpha_G \gg 1 \iff \Lambda_{\text{UV}} = M_G ; \text{UV completion} \]

We explore to solve these problems by changing the 4D flat space to the 5D Randall-Sundrum type warped background.

1. This model can easily provide a natural suppression for the Yukawa couplings by a wave function localization.
2. UV completion is provided by a strong gravity. (cf. AdS/CFT)
Realistic Hybrid Inflation in 5D Orbifold SO(10) GUT

Takeshi Fukuyama\textsuperscript{a,b}, Nobuchika Okada\textsuperscript{c,d} and Toshiyuki Osaka\textsuperscript{a}

\textsuperscript{a} Department of Physics, Ritsumeikan University
\textsuperscript{b} Ritsumeikan Global Innovation Research Organization, Ritsumeikan University
\textsuperscript{c} Department of Physics, University of Maryland
\textsuperscript{d} Theory Division, KEK
2. Model Setup

• 5D is compactified on the orbifold $S^1 / (\mathbb{Z}_2 \times \mathbb{Z}_2')$
  Assign $(P,P')$ the bulk SO(10) gauge multiplet as in Table 1, only the PS gauge multiplet has zero mode and the bulk 5D N=1 SUSY SO(10) gauge symmetry is broken to 4D N=1 SUSY PS symmetry.
SO(10) brane  \[ y = 0 \]  PS brane  \[ y = \pi R/2 \]
<table>
<thead>
<tr>
<th>$(P, P')$</th>
<th>bulk field</th>
<th>mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(+, +)$</td>
<td>$V(15,1,1), V(1,3,1), V(1,1,3)$</td>
<td>$\frac{2n}{R}$</td>
</tr>
<tr>
<td>$(+, -)$</td>
<td>$V(6,2,2)$</td>
<td>$\frac{(2n + 1)}{R}$</td>
</tr>
<tr>
<td>$(-, +)$</td>
<td>$\Phi(6,2,2)$</td>
<td>$\frac{(2n + 1)}{R}$</td>
</tr>
<tr>
<td>$(-, -)$</td>
<td>$\Phi(15,1,1), \Phi(1,3,1), \Phi(1,1,3)$</td>
<td>$\frac{(2n + 2)}{R}$</td>
</tr>
</tbody>
</table>

Table 1
<table>
<thead>
<tr>
<th>Matter Multiplets</th>
<th>Brane at $y=\pi R/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_i = F_{Li} \oplus F_{Ri}^c$ ($i = 1,2,3$)</td>
<td></td>
</tr>
</tbody>
</table>

| Higgs Multiplets | $\Psi = \frac{(1,2,2)_H, (1,2,2)'_H, (15,1,1)_H, (6,1,1)_H, (4,1,2)_H, (4,1,2)_H, (4,2,1)_H, (4,2,1)_H,}{3,2,1}(\psi_3 \otimes \psi_2 \otimes \psi_1)$ |

Table 2
Figure 1:
3. Smooth Hybrid Inflation

\[ W = S \left( -\mu^2 + \frac{(\bar{\phi}\phi)^2}{M^2} \right) \]

\[ V = \left( -\mu^2 + \frac{(\bar{\phi}\phi)^2}{M^2} \right)^2 + 4S^2 \frac{|\phi|^2|\bar{\phi}|^2}{M^4} \left( |\phi|^2 + |\bar{\phi}|^2 \right) \]

\[ |\phi| = |\bar{\phi}| = \frac{\chi}{2}, \quad |S| = \frac{\sigma}{\sqrt{2}} \]

\[ V = \left( \mu^2 - \frac{\chi^4}{16M^2} \right)^2 + \frac{\chi^6\sigma^2}{16M^4} \]
Inflation trajectory

\[ \chi^2 = -6\alpha V \simeq \mu^4 \left( 1 - \frac{2v_{PS}^4}{27\sigma^4} \right) \] for \( \sigma^2 \gg v_{PS}^2 \gg v_{PS}^2 \)

\[ v_{PS} = \sqrt{\mu M} = 1.2 \times 10^{16} \text{GeV} \]
Slow-roll parameters

$$
\epsilon = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \simeq \frac{32 M_P^2 v_{PS}^8}{729 \sigma^{10}} \simeq -\frac{4 v_{PS}^4}{135 \sigma^4} \eta
$$

$$
\eta = M_P^2 \left( \frac{V''}{V} \right) = -\frac{40 M_P^2 v_{PS}^4}{27 \sigma^6}
$$

$$
\xi^2 = M_P^4 \left( \frac{V' V'''}{V^2} \right) \simeq \frac{640 M_P^4 v_{PS}^8}{243 \sigma^{12}}
$$

Where the prime denotes derivative with respect to $t \sigma$

**Slow-roll condition**

$$
\epsilon, |\eta| << 1
$$

**The end of inflation**

$$
|\eta| = 1
$$

$$
\sigma_f^6 = \frac{40}{27} M_P^2 v_{PS}^4
$$
Spectral index

\[ n_s \approx 1 - 6\varepsilon + 2\eta \]

Ratio of tensor-to-scalar fluctuations

\[ r \approx 16\varepsilon \]

Running of the Spectral index

\[ \alpha_s = \frac{dn_s}{d\ln k} \approx 16\varepsilon\eta - 24\varepsilon^2 - 2\xi^2 \]
The number of e–folds after the comoving scale $b_0 = 2\pi/k_0$ has crosses the horizon is given by

$$N_k = \frac{1}{M_P^2} \int_{\sigma_f}^{\sigma_k} d\sigma \frac{V}{V'} \approx \frac{9}{16 M_P^2 \nu_{PS}^4} (\sigma_k^6 - \sigma_f^6)$$

$\sigma_k^6 \gg \sigma_f^6$

$$N_k \approx -\frac{5}{6\eta_k}, \quad n_s \approx 1 - \frac{5}{3 N_k}, \quad \alpha_s \approx -\frac{5}{3 N_k^2}$$
The number of e-folds required for solving the horizon and flatness problems of the standard big bang cosmology is given by

\[ N_k \simeq 51.4 + \frac{2}{3} \ln \left( \frac{V(\sigma_k)^{1/4}}{10^{15} \text{ GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_{rh}}{10^7 \text{ GeV}} \right) \]

for \( k_0 = 0.002 \text{ Mpc}^{-1} \)

The power spectrum of the primordial curvature perturbation at the scale is given by

\[ P_R^{1/2} \simeq \frac{1}{2\sqrt{3\pi} M_P^3 |V'|} \frac{V^{3/2}}{16\sqrt{3\pi}} \frac{\sigma_k^5}{M_P^3 M^2} \]

WMAP 5-year data

\[ P_R \simeq 2.457 \times 10^{-9} \]
4. Numerical Results

Analysis 1

• Independent free parameters
  \[ M, \sigma_k, T_{rh} \]

• Fixed value
  \[ \nu_{PS} = 1.2 \times 10^{16} \text{ GeV} \]

• Analysis range
  \[ 1 \text{ MeV} \leq T_{rh} \leq 10^7 \text{ GeV} \]
Our results

\[ 0.963 \leq n_s \leq 0.968, \]
\[ 4.0 \times 10^{-7} \geq r \geq 3.1 \times 10^{-7}, \]
\[ -8.4 \times 10^{-4} < \alpha_s < -6.1 \times 10^{-4} \]

WMAP 5-year data

\[ n_s = 0.960^{+0.014}_{-0.013} \]
\[ r < 0.2 (95\% CL) \]
\[ \alpha_s = -0.032^{+0.021}_{-0.020} (68\% CL) \]

(consistent with zero in 95\% CL)
Analysis 2

- Independent free parameters
  \[ M, \sigma_k, \nu_{PS} \]
- Fixed value
  \[ T_{rh} = 10^7 \text{ GeV} \]
- Analysis range
  \[ \nu_{PS} \leq M \leq M_P \]
We obtain

\[1.9 \times 10^{14} \text{ GeV} \leq v_{PS} \leq 5.6 \times 10^{16} \text{ GeV}\]
\[ M \sim M_5 \sim M_{\text{GUT}} = 4.6 \times 10^{17}\text{GeV} \]

\[ 0.967 \leq n_s \leq 0.968, \]
\[ -6.4 \times 10^{-4} \leq \alpha_s \leq -6.1 \times 10^{-4} \]
5. Conclusions

• We have discussed the smooth hybrid inflation scenario in the context of the SO(10) GUT model.

• The running gauge coupling fixes the model parameters as $M_{GUT} = 4.6 \times 10^{17}$ and $V_{PS} = 1.2 \times 10^{16} GeV$

These parameters are determined independently of the cosmological considerations, nevertheless the inflation scenario with these parameters fits very well with the current cosmological observations,