Unkonventionelle Supraleitung

Serie 7

Verteilung: 13.Dezember Abgabe: 20.Dezember

7.1 Here we discuss the theory of the vibrating-wire viscometer used in the measurements of the viscosity of the liquid ^{3}He. Consider a thin wire of length l along the x axis, mass per unit length μ, fixed at both ends and subjected to a tension T. The wire is immersed in a viscous fluid. A uniform and time independent magnetic field B is applied perpendicular to the wire ($\mathbf{B} \parallel \hat{z}$). The wire carries a current of the form $I_0 e^{i\omega t}$. The wire then performs small oscillations in the x-y plane.

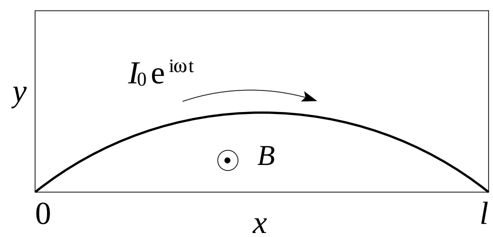

Figure 1: Schematic figure of the system.

Consider the curve $y(x,t)$ describing the motion of the wire. As a boundary condition, $y(0,t) = y(l,t) = 0$. The equation of motion for a small element of the wire is given as

$$\mu \frac{\partial^2 y}{\partial t^2} = F,$$

where the force F per unit length is composed of the elastic F_e, the viscous F_v, and the magnetic F_m:

$$\begin{aligned} F &= F_e + F_v + F_m, \\ F_e &= T\frac{\partial^2 y}{\partial x^2}, \\ F_v &= -D\frac{\partial y}{\partial t} - \mu_L \frac{\partial^2 y}{\partial t^2}, \\ F_m &= -I_0 B e^{i\omega t}. \qquad (\mathbf{F}_m = \mathbf{I} \times \mathbf{B}) \end{aligned}$$

Here, D is a friction coefficient related to the viscosity of the fluid, and μ_L represents the effective mass of the fluid dragged by the moving wire (G. Stokes, 1901).[1] For simplicity, we treat D and μ_L as constant parameters here.

a) First, assume $I_0 = 0$, and consider the following oscillation which satisfies the boundary condition $y(0,t) = y(l,t) = 0$, (n is the integer):

$$y(x,t) = \sin\Big(\frac{n\pi x}{l}\Big) e^{i\tilde{\omega} t}.$$

[1] J. T. Tough, W. D. McCormick, and J. G. Dash, Rev. Sci. Instrum. **35**, 1345 (1964).

Show that in this case, $\tilde{\omega}$ is given as

$$\tilde{\omega} = i\alpha \pm \omega_n,$$

where

$$\alpha = \frac{D}{2(\mu+\mu_L)}, \qquad \text{and} \qquad \omega_n = \sqrt{\frac{T}{\mu+\mu_L}\Big(\frac{n\pi}{l}\Big)^2 - \alpha^2}.$$

b) From now on, the current is applied, i.e., $I_0 \neq 0$. Let us consider the following solution of the equation of motion:

$$y(x,t) = \sum_n A_n \sin\Big(\frac{n\pi x}{l}\Big) e^{i\tilde{\omega}t},$$

where n is the positive integer, and we assume that A_n is independent of the time t.

Show, in this case, that the amplitude A_n is given by

$$A_n = \delta_{n,2j-1}\Big(\frac{4I_0B}{n\pi\mu}\Big)\frac{1}{\Big(1+\frac{\mu_L}{\mu}\Big)\omega^2 - \frac{T}{\mu}\Big(\frac{n\pi}{l}\Big)^2 - i\frac{D}{\mu}\omega}, \qquad (j = 1,\ 2,\ 3,\ \cdots)$$

and that under the above assumption (t-independence of A_n), the frequency $\tilde{\omega}$ of vibration of the wire must be identical to the frequency ω of the current, i.e., $\tilde{\omega} = \omega$.

By the way, from that result for the amplitude A_n, one can notice that owing to the viscosity of the fluid, (i) the resonance frequencies shift from Ω_n to $\Omega_n/\sqrt{1+\frac{\mu_L}{\mu}}$ and (ii) the resonances acquire a width of the order of D/μ. Here, $\Omega_n \equiv \sqrt{\frac{T}{\mu}}(\frac{n\pi}{l})$ are the resonance frequencies measured in vacuum without the viscous fluid, $(n = 1,\ 3,\ 5,\ \cdots)$. The resonance phenomena can be observed experimentally by an induced voltage between edges of the oscillating wire.

c) The movement of a conducting wire in a magnetic field induces a voltage between its edges. The induced voltage $V(t)$ between edges of a wire with length L is given by

$$V(t) = \int_0^L (\mathbf{v}(t)\times\mathbf{B})\cdot d\mathbf{l},$$

where $\mathbf{v}(t)$ is the velocity of the small line element $d\mathbf{l}$ of the wire.

Calculate $V(t)$ for the present oscillating wire when its dynamics is described by $y(x,t)$ of the problem **b)**.

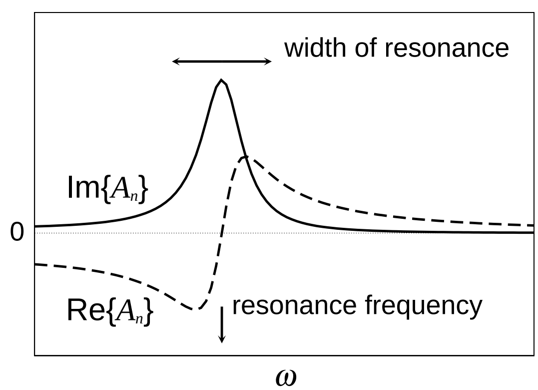

Figure 2: Schematic plot of A_n vs. ω.

Hint: One may utilize the following relations.

$$\int_0^l dx \sin\Big(\frac{m\pi x}{l}\Big)\sin\Big(\frac{n\pi x}{l}\Big) \;=\; \frac{l}{2}\delta_{m,n}.$$

(m and n are the integers.)

$$\begin{aligned}\int_0^l dx \sin\Big(\frac{n\pi x}{l}\Big) &= \begin{cases} \dfrac{2l}{n\pi} & (n=1,\ 3,\ 5,\ \cdots) \\ 0 & (n=2,\ 4,\ 6,\ \cdots)\end{cases} \\ &= \frac{2l}{n\pi}\delta_{n,2j-1}. \qquad (j\text{: the positive integer})\end{aligned}$$

7.2 Consider the following implicit equation for the depression of the critical temperature T_c in the presence of a pair breaking mechanism characterized by τ:

$$\ln\Big(\frac{1}{x}\Big) = \Psi\Big(\frac{1}{2}+\frac{y}{4\pi x}\Big) - \Psi\Big(\frac{1}{2}\Big),$$

which is equivalent to Eq. (3.19) given in the *German* version of the theory lecture notes and to Eq. (V.1) in the experiment lecture notes. Here, $x = T_{\rm c}/T_{\rm c0}$, $y = 2\hbar/\tau k_{\rm B}T_{\rm c0}$, and $T_{\rm c0}$ is the original critical temperature in the absence of pair breaking effects.

From the above equation, for each value of x ($0 \le x \le 1$), a unique value of $y = U_n(x)$ is determined. The function $U_n(x)$ is called "universal" in the sense that it contains only dimensionless and material-independent parameters. Unfortunately, there is no analytical expression for $U_n(x)$, and only numerical values of $U_n(x)$ can be obtained for general x.

Here let us consider the following two limiting situations alternatively.

a) Show that for $x \to 0$ ($T_{\rm c} \to 0$), $y = U_n(x) \to 1.76$, and $\frac{\hbar}{\tau} \sim k_{\rm B}T_{\rm c0}$.

b) Show that for $x \to 1$ ($T_{\rm c} \to T_{\rm c0}$), $y = U_n(x) \to (8/\pi)(1-x)$, and $\frac{\hbar}{\tau} \sim k_{\rm B}(T_{\rm c0} - T_{\rm c})$.

$\hbar/\tau$ represents the strength of a pair breaking effect.

Hint: One may utilize the following approximations for the digamma function Ψ given in Eq. (3.20),

$$\Psi\Big(\frac{1}{2}+z\Big) \approx \Psi\Big(\frac{1}{2}\Big) + \frac{\pi^2}{2}z \qquad (z \to 0),$$
$$\Psi(z) \approx \ln z - \frac{1}{2z} - \frac{1}{12z^2} \qquad (z \to \infty),$$

and $\Psi(\frac{1}{2}) = -\ln(4 \times e^{\gamma}) \approx -\ln(4 \times 1.78) \approx -1.96$, ($\gamma \approx 0.5772$).