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Near-degeneracy of extended $s + d_{x^2−y^2}$ and $d_{xy}$ order parameters in quasi-two-dimensional organic superconductors

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The symmetry of the superconducting order parameter in quasi-two-dimensional bis-ethylenedithio-tetrathiafulvalene (BEDT-TTF) organic superconductors is a subject of ongoing debate. We report ab initio density-functional-theory calculations for a number of organic superconductors containing $κ$-type layers. Using projective Wannier functions, we derive the parameters of a common low-energy Hamiltonian based on individual BEDT-TTF molecular orbitals. In a random-phase approximation spin-fluctuation approach, we investigate the evolution of the superconducting pairing symmetry within this model, and we point out a phase transition between extended $s + d_{x^2−y^2}$ and $d_{xy}$ symmetry. We discuss the origin of the mixed order parameter and the relation between the realistic molecule description and the widely used dimer approximation. Based on our ab initio calculations, we position the investigated materials in the obtained molecule model phase diagram, and we simulate scanning tunneling spectroscopy experiments for selected cases. Our calculations show that many $κ$-type materials lie close to the phase-transition line between the two pairing symmetry types found in our calculation, possibly explaining the multitude of contradictory experiments in this field.

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I. INTRODUCTION

Quasi-two-dimensional organic charge-transfer salts of bis-ethylenedithio-tetrathiafulvalene molecules, abbreviated BEDT-TTF or ET, have attracted much interest due to their rich phase diagrams [1,2]. Among these materials, the family of $κ$-(ET)$_2$X salts, where $κ$ refers to a specific arrangement of the ET molecules and $X$ corresponds to a monovalent anion, stands out due to the realization of fascinating states of matter such as the Mott insulator, unconventional superconductors, or spin liquid [1–6]. In particular, the immediate vicinity of the superconducting phase to an antiferromagnetic Mott insulator suggests a deeper connection between two-dimensional organics and high-temperature cuprate superconductors [7].

Although superconducting $κ$-type charge-transfer salts have been investigated, for instance in studies of specific heat [3,8–12], surface impedance [13], thermal conductivity [14], millimeter-wave transmission [15], scanning tunneling spectroscopy (STS) [16–19], and elastic constants [20], no consensus about the symmetry of the superconducting pairing in this model has been reached so far. Some of the experiments are in favor of $s$-wave symmetry [3,8,9], while other studies have proposed $d$-wave states with contradictory positions of the nodes in the superconducting order parameter [10–18]. Evidence for a mixed-symmetry order parameter was recently provided in Refs. [19,20]. Notably, evidence for a phase separation between different $d$-wave states has recently been reported in Ref. [18].

In theoretical approaches, the $κ$-(ET)$_2$X family of materials is often described by a half-filled Hubbard model of (ET)$_2$ dimers on the anisotropic triangular lattice [21–25], which is equivalent to a square lattice model with an additional coupling along one of the diagonals. Many theoretical methods have been applied to the dimer-based Hubbard model, for instance the fluctuation-exchange approximation (FLEX) [26–29], the path-integral renormalization group [30], cluster dynamical mean-field theory [31–33], variational Monte Carlo [34–37], and exact diagonalization [38,39]. These studies do not agree entirely on all the details of the phase diagram, especially whether superconductivity is realized in the model or not. Those studies that do show superconductivity nevertheless agree, with the exception of Ref. [29], that it is of $d_{x^2−y^2}$-type as in high-temperature cuprate superconductors [40].

Although the triangular lattice Hubbard model has been remarkably successful in explaining the overall phase diagram [1,41,42] and also some more subtle physics [43] of $κ$-(ET)$_2$X materials, the recent discovery of multiferroicity [44] in antiferromagnetic $κ$-(ET)$_2$Cu[N(CN)$_2$]Cl and the still unresolved problem of superconducting-pairing symmetry [45,46] call for alternative approaches. In particular, the dimer model on the anisotropic triangular lattice is only an approximation with an a priori unclear range of applicability to the real lattice structure of $κ$-(ET)$_2$X charge-transfer salts. In a seminal paper [47], Kuroki et al. investigated superconducting pairing, taking into account the realistic lattice structure, and in fact they found a phase transition between $d_{x^2−y^2}$ and $d_{xy}$-symmetric states when lowering the degree of dimerization. Other possible directions of future theoretical research beyond the dimer Hubbard model are outlined in Refs. [48–51].

In this work, building upon the idea by Kuroki et al., we derive a set of realistic molecule-based low-energy models for superconducting $κ$-(ET)$_2$X materials from ab initio density-functional-theory (DFT) calculations. After identifying the parameter region relevant for the real materials, we investigate the symmetry of the superconducting pairing in this model within a random-phase approximation (RPA) spin-fluctuation approach. Our results show that the position of many materials in the phase diagram is close to a phase-transition line between states with extended $s + d_{x^2−y^2}$ and $d_{xy}$ pairing symmetry. Furthermore, we clarify that the customary dimer model not only fails in the limit of weak dimerization, but also when the in-plane anisotropy of hopping integrals becomes too large, which we find to be the case for all investigated materials.

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Finally, we simulate tunneling spectra in the superconducting state for selected cases, and we compare our findings to relevant experiments.

II. METHODS AND MODELS

A. Ab initio calculations and model Hamiltonian

We use ab initio density-functional-theory (DFT) calculations within an all-electron full-potential local orbital (FPLO) [52] basis to calculate the electronic band structure. For the exchange-correlation functional, we employ the generalized gradient approximation (GGA) [53]. All calculations are converged on $6 \times 6 \times 6$ $k$-point grids. We use crystal structures from Refs. [54–56]. In the case of Ref. [56], where crystal structures were measured for several temperatures, we use the data taken at 100 K.

In contrast to the customary dimer approximation, we model the \( \kappa \)-(ET)\(^2\) layer, taking into account each individual ET molecule as a lattice site (see Fig. 1). Tight-binding parameters are obtained from projective molecular orbital Wannier functions as implemented in the FPLO code [57]. Therefore, the number of bands in the tight-binding model is equal to the number of ET molecules in the crystallographic unit cell. With the molecular Wannier function method, almost perfect representations of the DFT band structures can be obtained, and ambiguities from fitting procedures are avoided. The latter is especially important for many-body calculations based on the obtained low-energy Hamiltonians.

In the following model investigation, we only keep the four largest in-plane hopping elements \( t_1, t_2, t_3, t_4 \) between ET molecules [see Fig. 1(a)]. The resulting hopping structure is a generalization of the Shastry-Sutherland lattice [58], which is reached in the limit of \( t_2 = t_4 \) and \( t_3 = 0 \). In cases in which the unit cell contains multiple \( \kappa \)-type layers, we discard all but one of the layers after the Wannierization procedure, because the interlayer coupling is negligible. In some of the investigated compounds, the crystal symmetry is lowered with respect to the high-symmetry orthorhombic space group \( \text{Pnma} \) of \( \kappa \)-(ET)\(^2\)Cu[N(CN)\(_2\)]Br, which leads to a small additional splitting of the hoppings \( t_i \) into \( t_i \) and \( t_i' \). For simplicity, this particular anisotropy is discarded in our study by averaging the hopping integrals as \( t_i = (t_i + t_i')/2 \). As a result, we obtain the kinetic part of a four-band Hamiltonian, which is \( 3/4 \)-filled and of the same form for all materials investigated,

$$ H_0 = \sum_{ij\sigma} t_{ij} (c_{i\sigma}^{\dagger} c_{j\sigma} + \text{H.c.}). $$

Alternatively, because ET molecules in a \( \kappa \)-type arrangement are quite strongly dimerized, it is popular to approximate the \( \kappa \)-(ET)\(^2\) layer by dimers on an anisotropic triangular lattice, integrating out the intradimer degrees of freedom. The parameters of this dimer model can be calculated directly from the molecular model using geometric formulas [21],

\[
\begin{align*}
  t &= (|t_2| + |t_4|)/2, \\
  t' &= |t_3|/2.
\end{align*}
\]

By convention, the dimer approximation uses the crystallographic unit cell containing two dimers [see Fig. 1(b)]. Therefore, the dimer-approximated Hamiltonian consists of two bands, which are half-filled. Note that based on the geometric formulas, any anisotropy between \( t_2 \) and \( t_4 \) of the molecule model is discarded when going from the molecule to the dimer model. With few exceptions [59], the dimer-approximated model nevertheless reproduces well the low-energy part of the original band structure. It has recently been demonstrated that improved estimates for dimer model parameters can be obtained by a Wannier function calculation [23,25,51].

The two-band dimer model can be unfolded to a one-band model by transforming to a unit cell of half the size and rotated by 45°. The so-obtained model is directly related to the square-lattice Hubbard model, but with an additional coupling along one of the diagonals. Results obtained in the one-band model are therefore rotated by 45° with respect to the physical Brillouin zone of organic charge-transfer salts, so that, e.g., different \( d \)-wave order parameters exchange their designation when going from one Brillouin zone to the other (see Fig. 2). Thus, the same physical order parameter that has \( d_{xy} \) symmetry in the realistic four-molecule–two-dimer unit cell [Fig. 2(b)] has \( d_{z^2−r^2} \) symmetry in the model one–dimer–one-unit cell [Fig. 2(c)]. In our study, we always work in the physical unit cell containing two dimers [Figs. 2(b) and 2(d)]. We refer to the small backfolded part of the Fermi surface close to the Brillouin zone boundary as the elliptic part of the Fermi surface, when we call those sheets running almost parallel to the \( k_z \) direction quasi-one-dimensional.

An overview of the unit cell and hopping paths for the molecule and dimer model is shown in Fig. 1. The resulting Hamiltonians in orbital space for all three cases are listed in Appendix A.

B. RPA spin-fluctuation calculations

In \( \kappa \)-(ET)\(_2\)X materials, there is strong evidence for antiferromagnetic spin fluctuations [60]. Therefore, we investigate the superconducting state of these materials based on a RPA spin-fluctuation approach in the singlet channel [61,62]. We
have extended our implementation from single-site multiorbital models [63,64] to multi-site single-orbital models relevant for the materials discussed here. Compared to the FLEX approximation used in Ref. [47], our RPA method uses only states at the Fermi level and neglects the electronic self-energy correction. While this approximation prevents us from making quantitative statements about the superconducting transition temperature \( T_c \), it reduces significantly the numerical cost compared to FLEX, so that we can calculate the momentum structure of the superconducting order parameter for numerous input parameter sets and with high angular resolution. Competing magnetically ordered or paramagnetic Mott insulating states are not investigated in our study. Furthermore, we do not investigate possible time-reversal symmetry-breaking superconducting states or spin-triplet pairing.

The low-energy Hamiltonian is given by the kinetic part \( H_0 \), derived with the Wannier-function method described above, and the intraorbital Hubbard interaction \( H_{\text{int}} \),

\[
H = H_0 + H_{\text{int}} = \sum_{ij\sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) + \frac{U}{2} \sum_i n_{i\sigma} n_{i\bar{\sigma}}.
\]  

Here, \( \sigma \) represents the spin and \( n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma} \). The sum over \( i \) runs over all ET sites in the unit cell. The interaction strength \( U \) is treated as a parameter. Note that the Coulomb repulsion on a dimer and the Coulomb repulsion on a molecule are not identical. At present, the role of intermolecular Coulomb repulsion is particularly unclear. The investigation of interaction terms beyond on-site repulsion is left for future studies.

We calculate the noninteracting static susceptibility \( \chi^0 \), where matrix elements \( a_{i\mu}(\vec{k}) \) resulting from the diagonalization of the initial Hamiltonian \( H_0 \) connect the orbital and the band space denoted by indices \( i \) and \( \mu \), respectively. The \( E_\mu \) are the eigenvalues of \( H_0 \) and \( f(E) \) is the Fermi function. \( N \) is the number of sites in the unit cell,

\[
\chi^0_{l\mu,\nu}(\vec{q}) = -\frac{1}{N} \sum_{\vec{k},\mu,\nu} a_{i\mu}^*(\vec{k}) a_{i\nu}^*(\vec{k}+\vec{q}) a_{i\nu}(\vec{k}+\vec{q}) a_{i\mu}(\vec{k}) \times \frac{f(E_\nu(\vec{k}+\vec{q})) - f(E_\mu(\vec{k}))}{E_\nu(\vec{k}+\vec{q}) - E_\mu(\vec{k})}.
\]

In our calculation, both \( \vec{q} \) and \( \vec{k} \) run over uniform grids spanning the reciprocal unit cell. Temperature enters the calculation through the Fermi functions.

The fraction in Eq. (4) becomes problematic in numerical calculations, when the band energies \( E_\mu \) become degenerate. However, the expression can be rectified using l’Hospital’s rule, which we use in practice when the magnitude of the denominator falls below a certain threshold (e.g., \( 10^{-7} \) eV). Here, \( \beta \) denotes the inverse temperature \( \beta = (k_BT)^{-1} \),

\[
\lim_{E_\nu \rightarrow E_\mu} \frac{f(E_\nu(\vec{k}+\vec{q})) - f(E_\mu(\vec{k}))}{E_\nu(\vec{k}+\vec{q}) - E_\mu(\vec{k})} = -\beta \frac{e^{\beta E_\nu}}{(e^{\beta E_\nu} + 1)^2}.
\]

The static spin and orbital susceptibilities (\( \chi_{s,\text{RPA}} \) and \( \chi_{c,\text{RPA}} \)) are constructed in an RPA framework. Since the interaction term defined in Eq. (3) is local and we have only one orbital per lattice site, we can restrict the calculation to the diagonal elements of the susceptibility and use scalar equations for the RPA-enhanced susceptibilities,

\[
\chi_{\ell}^{s,\text{RPA}}(\vec{q}) = \frac{\chi^0_{\ell}(\vec{q})}{1 - U \chi^0_{\ell}(\vec{q})},
\]

\[
\chi_{\ell}^{c,\text{RPA}}(\vec{q}) = \frac{\chi^0_{\ell}(\vec{q})}{1 + U \chi^0_{\ell}(\vec{q})}.
\]

Here, \( \chi_{\ell,\mu} \) with \( L = \{lll\} \) denotes the diagonal element of the susceptibility tensor associated with an ET site indexed by \( l \). Note that this formulation allows us to treat multiple inequivalent ET sites in the unit cell, keeping the individual \( \vec{q} \) dependence of their associated susceptibilities. Therefore, the symmetry of the susceptibility follows the symmetry of the ET layer in the crystallographic unit cell, which is important for checking the simplified four-parameter model against \textit{ab initio} Hamiltonians, which can have monoclinic symmetry, as, e.g., in \( \kappa-(ET)_2\text{Cu(NCS)}_2 \), or even triclinic symmetry, as in \( \kappa'-\text{Ag(CF}_3\text{)}_3\text{TC}(\text{TCE}) \).

The total spin susceptibility is given by the sum over all site-resolved contributions,

\[
\chi^s(\vec{q}) = \frac{1}{2} \sum_{\ell} \chi_{\ell}^{s,\text{RPA}}(\vec{q}).
\]
The pairing vertex in orbital space for the spin-singlet channel can be calculated using the fluctuation exchange approximation [65,66],
\[
\Gamma_{\mu\nu}(\vec{k},\vec{k}') = \text{Re} \sum_{l_1l_2l_3l_4} a^{l_1\mu}_{\nu}(\vec{k}) a^{l_1\nu}_{\mu}(-\vec{k}) [\Gamma_{l_1l_2l_3l_4}(\vec{k},\vec{k}')] \times a^{l_2\mu}_{\nu}(\vec{k}') a^{l_3\nu}_{\mu}(-\vec{k}').
\]
\[
\Gamma_{\mu\nu}(\vec{k},\vec{k}') = \text{Re} \sum_{l_1l_2l_3l_4} a^{l_1\mu}_{\nu}(\vec{k}) a^{l_1\nu}_{\mu}(-\vec{k}) [\Gamma_{l_1l_2l_3l_4}(\vec{k},\vec{k}')] \times a^{l_2\mu}_{\nu}(\vec{k}') a^{l_3\nu}_{\mu}(-\vec{k}').
\]
\[
-\sum_{\nu} \int_{C_{\nu}} dk' \frac{1}{2\pi v_F(k')} [\Gamma_{\mu\nu}(\vec{k},\vec{k}')] g_\nu(\vec{k}) = \lambda_i g_\nu(\vec{k}).
\]
\[
H = \sum_{k,\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k'} U(k,k') c_{k\sigma}^\dagger c_{-k\sigma}^\dagger c_{-k'-1\sigma} c_{k'-1\sigma}.
\]
The interaction can be treated in mean-field theory \[\delta(c^{\dagger} c^{\dagger}) = c^{\dagger} c^{\dagger} - \langle c^{\dagger} c^{\dagger} \rangle\], where terms quadratic in \(\delta\) are neglected. The resulting Hamiltonian can be diagonalized using the Bogoliubov-Valatin transformation, which introduces quasiparticle creation and annihilation operators \(\gamma^{\dagger}_{k\sigma} \) and \(\gamma_{k\sigma}\). The quasiparticle excitation energies are given as \(E_k = \sqrt{\epsilon_k^2 + |\Delta|^2}\), where \(\Delta(k) = \sum_k U(k,k')\langle c_{-k\sigma} c_{k'\sigma}\rangle\).

The BCS Hamiltonian can be rewritten in terms of the quasiparticle creation and annihilation operators,
\[
H_{\text{BCS}} = \sum_{k,\sigma} E_k \gamma^{\dagger}_{k\sigma} \gamma_{k\sigma} + \sum_k \epsilon_k - \sum_{k,k'} U(k,k')\langle c_{-k\sigma} c_{k'\sigma}\rangle.
\]

The excitation spectrum of the quasiparticles \(E_k\) is gapped and defined only for positive energies. The density of states of quasiparticles in an isotropic s-wave superconductor can be calculated from the normal-state density of states \(\rho(\epsilon)\) and the constant superconducting gap \(\Delta\).

\[
\rho_{\text{qp}}(E) = \frac{1}{N} \sum_k \delta(E - E_k) = \int d\epsilon \rho_0(\epsilon) \sqrt{\epsilon^2 + |\Delta|^2} \delta(\epsilon - \sqrt{\epsilon^2 + |\Delta|^2})
\]
\[
= \int d\epsilon \frac{\rho_0(\sqrt{E^2 - |\Delta|^2})}{\sqrt{E^2 - |\Delta|^2}} E > |\Delta|, \quad 0, \quad E < |\Delta|.
\]

The previous derivation assumed an isotropic gap and an energy dispersion of free electrons to identify the normal-state DOS \(\rho_0\). For realistic electronic structure and anisotropic gap \(\Delta\) this factorization of contributions is not easily possible due to the nontrivial momentum dependence of both functions:

\[
\rho_{\text{qp}}(E) = \int d\epsilon \frac{1}{N} \sum_k \delta(\epsilon - \epsilon_k) \delta(|E| - \sqrt{\epsilon^2 + |\Delta|^2})
\]
\[
\neq \int d\epsilon \rho_\gamma(\epsilon) \delta(|E| - \sqrt{\epsilon^2 + |\Delta|^2}).
\]

C. Simulation of tunneling spectra in the superconducting state

The central quantity measured in the scanning tunneling spectroscopy (STS) experiments on superconductors is the local density of states (DOS) in the superconducting phase. Here we start from the standard Bardeen-Cooper-Schrieffer (BCS) theory for isotropic s-wave superconductors. A simple approximate extension allows us to treat realistic Fermi surfaces and unconventional pairing symmetries derived from the \textit{ab initio} calculations combined with RPA spin-fluctuation theory as presented above.

To derive an approximation for the DOS of a superconductor, we start with the Hamiltonian for Cooper pairs with vanishing total momentum \[67,\]

\[
H = \sum_{k,\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k'} U(k,k') c_{k\sigma}^\dagger c_{-k\sigma}^\dagger c_{-k'-1\sigma} c_{k'-1\sigma}.
\]

We introduce in this expression a finite quasiparticle lifetime \[70,\] by adding an imaginary part \(\Gamma\) to the quasiparticle excitation energies. This allows us to carry out calculations with finite angular resolution and facilitates comparison to experiment. Furthermore, we improve upon the circular integration by replacing it with a summation over the discretized realistic Fermi surface, and we drop the irrelevant prefactors.
to obtain the final expression for the quasiparticle DOS in our study:

$$\rho_{qp}(E) \propto \sum_{\vec{k}} \text{Re} \frac{|E + i\Gamma|}{\sqrt{(E + i\Gamma)^2 - \Delta(\vec{k})^2}}. \quad (16)$$

In this form, the connection to the ab initio and RPA spin-fluctuation calculations is easily obtained: the vectors $\vec{k}$ in Eq. (16) all lie on the Fermi surface determined from the ab initio derived tight-binding model, and the gap $\Delta(\vec{k})$ on the Fermi surface can be substituted by the symmetry function $g_i(\vec{k})$ extracted from the RPA [Eq. (10)]. Note that the overall energy scale of the superconducting gap is not included in $g_i(\vec{k})$ because our formalism neglects the electronic self-energy and lacks a self-consistency condition. We checked that our approximation agrees well with a direct calculation of the quasiparticle spectrum based on Eq. (12), which is numerically more costly.

The quasiparticle DOS $\rho_{qp}(E)$ corresponds to the local density of states (LDOS) measured in STS experiments. For a direct comparison, thermal smearing and additional effects such as electronic disorder might have to be taken into account [19,43,71].

III. RESULTS AND DISCUSSION

A. Ab initio calculations

Using ab initio density-functional-theory calculations and subsequent Wannier downfolding, we determine the parameter sets $(t_1,t_2,t_3,t_4)$ corresponding to superconducting $\kappa$-(ET)$_2$X materials with anions $X \in \{\text{Ag(CF}_3)_4(\text{TCE}), \text{I}_3, \text{Ag(CN)}_2 \cdot \text{H}_2\text{O}, \text{Cu(NCS)}_2, \text{Cu[N(CN)}_2]\text{I(CN), Cu[N(CN)}_2]\text{Br}\}$, as well as polymorphs $\kappa$-$\alpha_1'$-(ET)$_2$Ag(CF$_3$)$_4$(TCE) and $\kappa$-$\alpha_2$-(ET)$_2$Ag(CF$_3$)$_4$(TCE), which also contain charge-ordered insulating $\alpha'$-type layers. The calculated parameters are listed in Table I. In the case of (ET)$_2$Ag(CF$_3$)$_4$(TCE) polymorphs (TCE abbreviates 1,1,2-trichloroethane), we rely on a previous ab initio calculation with an identical setup [59]. The small asymmetry of hoppings due to the lowered symmetry in these materials is averaged out to obtain a four-parameter model. For the original models, see Ref. [59].

We observe that all materials fall into a narrow region of parameters: $t_1 \in [165,190]$ meV, $t_2 \in [95.6,119]$ meV, $t_3 \in [52.2,82.4]$ meV, and $t_4 \in [17.3,36.3]$ meV. Normalizing $t_2$, $t_3$, and $t_4$ with respect to $t_1$, this means all materials lie in the range $t_2/t_1 \in [0.538,0.661]$, $t_3/t_1 \in [0.289,0.404]$, and $t_4/t_1 \in [0.099,0.220]$. Note the pronounced anisotropy between $t_2$ and $t_4$. These intervals of $t_2/t_1$, $t_3/t_1$, and $t_4/t_1$ obtained from the ab initio calculations determine the parameter ranges for our following model investigation.

We sorted the materials according to their superconducting transition temperature $T_c$, but we found no correlation of $T_c$ with either $t_1$, $t_2$, $t_3$, or $t_4$. The ratios $t_2/t_1$, $t_3/t_1$, or $t_4/t_1$ are also not obviously connected to $T_c$.

B. Pairing symmetry in the dimer model

First, we apply the RPA spin-fluctuation formalism to the dimer model in the range $t'/t \in [0,1]$. We evaluate the superconducting order parameter in fine steps of $t'/t$ and compare the leading eigenfunctions. In all cases, we find that a $d_{yy}$ state is the leading pairing symmetry.

Relating the dimer model back to the one-band model explained in Sec. II, the $d_{yy}$ state we find is identical to the $d_{x^2-y^2}$ state of the square-lattice Hubbard model after unfolding the Brillouin zone (see Fig. 2). Typical superconducting $\kappa$-(ET)$_2$X materials lie in the region $t'/t \lesssim 0.65$ [23,42,43], where a $d_{yy}$ solution is to be expected, as the dimer model is basically a square lattice of hoppings $t$, perturbed by the additional diagonal coupling $t'$. The diagonal coupling $t'$ breaks the $C_4$ symmetry of the underlying square lattice and gives the Fermi surface its elliptic shape, but the dominant terms in the Hamiltonian remain square-lattice-like. For a full account of possible pairing symmetries in the one-band Hubbard model on the square lattice, see Ref. [75].

An early theoretical study of the antiferromagnetic phase of $\kappa$-type materials concluded that the two molecules within a dimer carry the same spin and that the spins are flipped between neighboring dimers [22], giving rise to $(\pi,\pi)$ magnetic order in the parent compounds of high-temperature cuprate superconductors [40]. This result is consistent with our observation that a dimerized model gives a $d_{yy}$ order parameter in the physical Brillouin zone, which becomes a $d_{x^2-y^2}$ symmetry in the unfolded zone of the one-band model (see Fig. 2), again emphasizing the deep connection between cuprates and quasi-two-dimensional organic superconductors.

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$ (K)</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_4$</th>
<th>$t_2/t_1$</th>
<th>$t_3/t_1$</th>
<th>$t_4/t_1$</th>
<th>$t_2/t_1$</th>
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<tbody>
<tr>
<td>(ET)$_2$Ag(CF$_3$)$_4$(TCE)</td>
<td>2.6</td>
<td>168</td>
<td>102</td>
<td>60.8</td>
<td>33.4</td>
<td>0.610</td>
<td>0.362</td>
<td>0.199</td>
<td>0.362</td>
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<tr>
<td>(ET)$_2$I$_3$</td>
<td>3.6</td>
<td>180</td>
<td>119</td>
<td>52.2</td>
<td>31.7</td>
<td>0.661</td>
<td>0.289</td>
<td>0.176</td>
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<tr>
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<td>104</td>
<td>60.4</td>
<td>23.6</td>
<td>0.567</td>
<td>0.326</td>
<td>0.173</td>
<td>0.305</td>
</tr>
<tr>
<td>(ET)$_2$Ag(CF$_3$)$_4$(TCE)</td>
<td>9.5</td>
<td>166</td>
<td>97.6</td>
<td>65.8</td>
<td>35.3</td>
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<td>0.396</td>
<td>0.213</td>
<td>0.362</td>
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<tr>
<td>(ET)$_2$Cu(NCS)$_2$</td>
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<td>190</td>
<td>102</td>
<td>82.4</td>
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<td>0.538</td>
<td>0.387</td>
<td>0.092</td>
<td>0.171</td>
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<td>165</td>
<td>98.4</td>
<td>66.7</td>
<td>36.3</td>
<td>0.596</td>
<td>0.404</td>
<td>0.220</td>
<td>0.369</td>
</tr>
<tr>
<td>(ET)$_2$Cu[N(CN)$_2$]I</td>
<td>11.2</td>
<td>175</td>
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<td>78.5</td>
<td>17.3</td>
<td>0.574</td>
<td>0.344</td>
<td>0.099</td>
<td>0.172</td>
</tr>
<tr>
<td>(ET)$_2$Cu[N(CN)$_2$]Br</td>
<td>11.6</td>
<td>177</td>
<td>95.6</td>
<td>60.0</td>
<td>36.2</td>
<td>0.541</td>
<td>0.339</td>
<td>0.205</td>
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</tbody>
</table>

TABLE I. Values of the molecule model parameters $(t_1,t_2,t_3,t_4)$, also commonly denoted as $(b_{1p},b_{2q},q)$, for selected superconducting $\kappa$-(ET)$_2$X materials. All values are given in meV. Hopping integrals for the (ET)$_2$Ag(CF$_3$)$_4$(TCE) family are averages of the parameters given in Ref. [59], where we used the same method and settings to calculate the parameters as in the present study. Crystal structures for these materials were taken from Refs. [54,55]. All other crystal structures are taken from Ref. [56]. The values for $T_c$ are taken from Refs. [56,72–74].
We would also like to note that the authors of Ref. [18], referring to the physical Brillouin zone, invoked a $d_{xy}$ superconducting symmetry close to insulating patches and a $d_{x^2-y^2}$ state in the bulk to explain the findings of their STS study on deuterated $\kappa$-(ET)$_2$Cu[N(CN)$_2$]Br. As the antiferromagnetic insulating state is dimerized (see Ref. [22]), the dimer approximation naturally applies and gives a $d_{xy}$ order parameter in accordance with the experimental observation. What remains to be answered in an approach beyond the dimer model, as presented in the next sections, is why the superconducting order parameter of the bulk is $d_{x^2-y^2}$.

C. Pairing symmetry in the molecule model

The obvious step for going beyond the dimer model is to use the original crystal lattice, i.e., the molecule model explained above (see Fig. 1). To compare the results of the molecule model to those of the dimer model, we make the following consideration: via the geometric Eq. (2), the four-parameter molecule model is mapped onto a two-parameter dimer model. With this procedure we are left with two adjustable parameters in the molecule model whose variation discloses important features of the systems, not captured in the resulting dimer model, which remains unchanged. These adjustable parameters are the degree of dimerization $t_1/\max(t_2,t_3,t_4)$ and the in-plane anisotropy $t_4/t_2$.

The degree of dimerization obviously decides whether the dimer approximation applies to a material or not. Its influence on the superconducting pairing was quantified by Kuroki et al., who found a transition to a antiferromagnetic insulating state is dimerized (see Ref. [22]), in the dimer model [compare Figs. 3(d) and 3(e)]. Upon lowering $t_4/t_2$, the maxima of the superconducting gap shift toward the position where the nodes in a $d_{xy}$-symmetric state are and additional nodes appear on the quasi-one-dimensional part of the Fermi surface close to $(\pm \pi,0)$. As this shift is equivalent to a rotation by 45°, the state with eight nodes can be expected to have a significant $d_{x^2-y^2}$ contribution. In the limit of $t_4 \ll t_2$, the additional set of nodes on the quasi-one-dimensional part of the Fermi surface vanishes [see Fig. 3(a)]. The remaining four nodes are situated close to the Brillouin-zone boundary, where the smaller elliptic part of the Fermi surface is folded back. The details of the pairing symmetry are discussed further below. In what follows, we investigate the origin of the gap maxima shifts.

As an example, we show in Fig. 4 how extrema of the gap magnitude with opposite sign appear where parts of the Fermi surface can be connected by a wave vector $\vec{q}$ that shows a peak in the spin susceptibility. Note that in Fig. 4 the Brillouin zone is shifted by a vector $(\pi,\pi)$, because the relevant vectors $\vec{q}$ connect pieces of the Fermi surface across the boundaries of the Brillouin zone used in Fig. 3.

Now we come back to the discussion of the results presented in Fig. 3. At $t_4/t_2 = 0$, peaks appear at $\vec{q}_1 \approx (\pm 0.7\pi,0)$ and $\vec{q}_2 \approx (\pm \pi/2,\pm \pi)$, the dominant contribution to the spin susceptibility being the peak at $\vec{q}_2$. As $t_4/t_2$ is increased, the

![FIG. 3. Comparison of molecule models (a)–(d) with different ratios of $t_4/t_2$, which all correspond to the same dimer model (e) with $t'/t = 0.6$. The top panel shows the spin susceptibilities, where arrows $\vec{q}_1$ and $\vec{q}_2$ indicate the main features, while the bottom panel shows the leading eigenfunction of the superconducting gap equation on the Fermi surface. In the molecule models $t_1/t_2 = 0.333$ is fixed, while the ratio of $t_4/t_2$ is varied under the condition $t_1/(t_2 + t_4) = t'/t = 0.6$.](024515-6)
FIG. 4. (a) Spin susceptibility of the molecule model at $t_2/t_1 = 0.417$, $t_3/t_1 = 0.333$, and $t_4/t_1 = 0.139$, with arrows $\vec{q}_1$ and $\vec{q}_2$ indicating the main features. The data shown are the same as in Fig. 3(b). (b) Superconducting gap function on the Fermi surface for the same parameter values as in (a). Vectors $\vec{q}_1$ and $\vec{q}_2$ are the same as in (a) and connect parts of the Fermi surface with different signs of the gap. Note that the plot range of the Brillouin zone is shifted by $\pi$ toward $k_y \approx \pm\pi/2$, merely reflect the changing shape of the Fermi surface. For $t_3 \ll t_2$, the additional set of nodes on the 1D sheets vanishes, because they are no longer connected by $\vec{q}_2$, which now instead points from the 1D sheet to the elliptic parts just like $q_1$. This consideration shows that the pairing-symmetry transition in the molecule model is driven by a peculiar competition between $\vec{q}_1$ and $\vec{q}_2$ nesting vectors.

Now we connect the structure of the susceptibility and the superconducting pairing to the underlying lattice model. The feature at $\vec{q}_1$ is obviously connected to the $t_2$ hopping parameter, since it is the only hopping exclusively in the $x$ direction (compare Fig. 1). All other $t$ parameters can only be responsible for a four-peak structure, as they occur pointing along both diagonals of the physical unit cell. The influence of the competition between $t_2$, $t_3$, and $t_4$ on the feature at $\vec{q}_2$ is, however, hard to quantify directly. Therefore, we decompose the superconducting order parameter in terms of extended $s$- and $d$-wave basis functions $f_i$ appropriate for a square-lattice geometry. For each of the $d$-wave basis functions, we also take into account the associated extended $s$-wave function, because we expect that a significant extended $s$-wave component could mix with the $d$-wave states to accommodate the orthorhombicity of the model:

$$f_{t_1}(\vec{k}) = \cos k_x + \cos k_y,$$  \hfill (17a)

$$f_{d_{x^2-y^2}}(\vec{k}) = \cos k_x - \cos k_y,$$  \hfill (17b)

$$f_{d_{x+y}}(\vec{k}) = \sin k_y - \sin k_y,$$  \hfill (17c)

$$f_{d_{x-y}}(\vec{k}) = \sin k_x - \sin k_y,$$  \hfill (17d)

Rotated into the Brillouin zone of $\kappa$-type materials, gap functions $f_d$ and $f_s$ are to be expected from antiferromagnetic exchange along squarelike bonds ($t_2$, $t_4$), while $f_s$ and $f_{d_{x^2-y^2}}$ correspond to exchange paths along diagonal bonds ($t_3$); see Fig. 5.

We fit the pairing symmetries calculated from the RPA to a linear combination of the previously defined pairing symmetries, and we determine their relative contributions $c_i$,

$$\tilde{\mathbf{g}}(\vec{k}) = c_1 f_{t_1} + c_{d_{x^2-y^2}} f_{d_{x^2-y^2}} + c_{d_{x+y}} f_{d_{x+y}} + c_{d_{x-y}} f_{d_{x-y}}.$$  \hfill (18)

For the $d_{xy}$ state, we state $c_{d_{xy}} = 1$ and all other contributions zero, i.e., except for the not well-reproduced extended nodal

FIG. 5. Hopping structure in the molecule model for the $\kappa$-(ET)$_2$I$_3$ layer. The dark gray arrows indicate the connection between hopping parameters and the symmetry functions appearing in the solution of the superconducting gap equation.
region close to \((\pm \pi, 0)\), the dimer model and the molecule model at \(t_1/t_2 \lesssim 1\) are dominated by the square-lattice physics of \(t\) and \(t_2\), respectively. For the \(d_{z^2}^\pm\)-like solution at \(t_2 \ll t_2\) we find negligible contributions from \(f_{bd}\) and \(f_{bd}\), dominant \(f_{bd}\), and subdominant \(f_{d_{z^2}}\). For increasing \(t_4/t_2\), the ratio of coefficients \(c_{d_{z^2}}/c_{d_{z^2}}\) decreases, i.e., the square-lattice physics becomes dominant when the asymmetry between \(t_2\) and \(t_4\) is removed. Using the symmetry functions \(f_t\), all details of the superconducting gap in the \(d_{z^2}^\pm\)-like state including the additional nodes can be reproduced by Eq. (18).

Our findings provide a clear picture of the pairing competition in the molecule model: in the realistic region of parameters, where the dimerization measured by \(t_1/\max(t_2, t_3, t_4)\) and the anisotropy of \(t_2\) and \(t_4\) are finite, the competition of squarelike \((t_2, t_4)\) and diagonal \((t_3)\) hopping realizes a unique linear combination of functions \(f_{d_{z^2}}\) and \(f_{d_{z^2}}\) as the leading pairing symmetry. We refer to this linear combination as \(s \pm d_{z^2}\), or extended \(s + d_{z^2}\). The \(s\)-wave contribution is equivalent to the \(s\)\(_\pm\) pairing state believed to be realized in iron-based superconductors (see, e.g., Ref. [76]), and it has been overlooked entirely in the literature on quasi-two-dimensional organic charge-transfer salts. When the lattice becomes more squarelike \((t_2 \lesssim t_4)\), i.e., the molecule model approaches the dimer limit, the \(d_{xy}\) symmetry known from the dimer model takes over. In other words, in the context of realistic modeling of \(\kappa\)-type materials, the \(d_{xy}\) symmetry found in the dimer model \((d_{xy}^\pm\) in the unfolded one-band model) is mostly an artifact of the underlying approximation to the real lattice structure [Eq. (2)].

Finally, we checked our results obtained with the four-parameter molecule model against the original hopping structure obtained from projective Wannier functions, which includes longer-range processes. As expected, the differences induced by the distance cutoff and parameter averaging are negligible.

**D. Pairing symmetry phase diagram of the molecule model**

To complete our study of the pairing symmetry competition, we investigated the leading pairing symmetry of the molecule model as a function of \(t_2/t_1\), \(t_3/t_1\), and \(t_4/t_1\) in the range of parameters realized in actual superconducting \(\kappa\)-type materials.

In Fig. 6 we show the obtained phase diagram, which consists of a \(d_{xy}\)-symmetric phase at low \(t_2/t_1\) and \(t_3/t_1\), while the rest of the phase diagram shows an \(s \pm d_{z^2}\) state. The consecutively numbered symbols in Fig. 6 correspond to the position of real materials as listed in Table I within this phase diagram. As we scanned the phase diagram several times for different fixed \(t_2/t_1\), materials were sorted into the cut with the closest value of \(t_4/t_1\).

At low \(t_2/t_1\) the phase boundary is almost horizontal, i.e., independent of the precise value of \(t_2/t_1\). For larger values of \(t_2/t_1\), the model becomes more asymmetric with respect to \(t_2\) and \(t_4\), and a smaller diagonal coupling \(t_3\) is sufficient to drive the system into the \(s \pm d_{z^2}\) state. The size of the \(d_{xy}\)-symmetric region is obviously determined by the value of \(t_4/t_1\) as explained in the preceding section.

In the numerical calculations, we observed that the leading two pairing symmetries are almost degenerate in a broad parameter region. This is to be expected, because the \(s \pm d_{z^2}\) state emerges precisely as a compromise between two different nesting vectors, of which one rather fits to a pure \(d_{xy}\) symmetry. To clarify this degeneracy, we calculated the eigenvalues of the leading and subleading solutions of the gap equation at fixed \(t_2/t_1\) and \(t_1/t_1\) and varied \(t_4/t_1\) in the range [0, 1]. Figure 7 shows the eigenvalues of both possible pairing states as a function of the in-plane anisotropy \(t_2/t_1\). We observe a pronounced asymmetry: While the \(d_{xy}\) state is competitive even for low values of \(t_2/t_1\), the \(s \pm d_{z^2}\) quickly becomes irrelevant when approaching the isotropic case \((t_4/t_2 = 1)\).

Finally, based on our parameter estimates, all materials investigated lie in the \(s \pm d_{z^2}\) region of the phase diagram. Materials particularly close to the phase-transition line are \(\kappa\)-\((ET) \_I_{3}\), \(\kappa\)-\((ET) \_I_{3}\), Ag(CN)\(_2\) \cdot H\(_2\)O, and \(\kappa\)-\((ET) \_I_{3}\). Cu(NCN)\(_2\). These can be expected to realize the \(s \pm d_{z^2}\) order parameter with eight nodes. Evidence for eight-node mixed-symmetry superconductivity has recently been found in Ref. [19].

When materials are close to the phase-transition line, small changes of the hopping parameters might drive them into the \(d_{xy}\) state, which is always present as a subdominant pairing symmetry. For such local changes of parameters, for instance...
lattice defects [77,78] or disorder of molecular conformations could be responsible. In Ref. [43] we have shown that different conformations of ET molecules result in decidedly different pairing symmetries as a function of the in-plane anisotropy \( t_2/t_1 \). The isotropic case is realized for \( t_2/t_1 = 1 \). Only \( t_3 \) was varied. The other parameters were fixed to \( t_2/t_1 = 0.5 \) and \( t_3/t_1 = 0.33 \).

In Ref. [22], a square-lattice-like antiferromagnetic order was found for the insulating state of \( \kappa \)-type materials. Therefore, we expect significant competition between antiferromagnetism and \( d_{xy} \)-symmetric superconductivity, while the \( s + d_{x^2−y^2} \)-symmetric state is realized farther away from the magnetically ordered insulating phase. Within this picture, recent results by Oka et al. [18], who interpreted their experiment in terms of patches with a \( dx \) order parameter and a \( d_{xy} \)-symmetric bulk, can be qualitatively explained.

At this point, we would like to point out that most experimental studies assume a four-node \( d \)-wave order parameter upon data analysis, which excludes from the start the detection of the \( s \) component we found. In particular, the realization of the \( s + d_{x^2−y^2} \) state with eight nodes that lie along the diagonals and close to the crystallographic axes may explain the considerable disagreement in the experimental literature regarding the node positions.

E. Simulation of scanning tunneling spectroscopy

Most transport experiments on \( \kappa \)-(ET)\(_2\)X materials have proven to be difficult to interpret and could not resolve the symmetry of the superconducting pairing so far. However, recent improvements in sample preparation for low-temperature scanning tunneling spectroscopy (STS) experiments have allowed for progress toward a resolution of the superconducting order parameter [18,19,71].

Therefore, in this section we simulate tunneling spectra in the superconducting state for molecule model parameters \( t_2/t_1 = 0.4375, t_4/t_1 = 0.1 \), and various values of \( t_3/t_1 \): a four-node \( s_\pm + d_{x^2−y^2} \) state is obtained for \( t_3/t_1 = 0.5 \), an eight-node \( s_\pm + d_{x^2−y^2} \) state for \( t_3/t_1 = 0.3475 \), and \( d_{xy} \) for \( t_3/t_1 = 0.25 \). We employ the representation of the superconducting gap in terms of symmetry functions introduced in Eq. (18), which we multiply with a prefactor \( \Delta_0 = 10 \) meV to obtain a spectrum with a reasonable energy scale. The gap on the Fermi surface is then given by \( \Delta(\vec{k}) = \Delta_0 \tilde{g}(\vec{k}) \). We use this expression together with Eq. (16) to calculate the quasiparticle density of states \( \rho_{qp} \), which corresponds to the local density of states (LDOS) observed in STS experiments. The finite quasiparticle lifetime is modeled by \( \Gamma = 0.07 \) meV. In the \( d_{xy} \) case, we ignore the small anisotropy found in the RPA calculation.

In Fig. 8 we show (i) the obtained gap on the Fermi surface, (ii) the magnitude of the gap versus angle measured from the \( k_x \) direction, and (iii) the simulated tunneling spectrum for the three cases investigated.

The magnitude of the gap versus the angle is distributed anisotropically on the Fermi surface [Figs. 8(a)–8(c), top panel]. Maxima of the gap magnitude are indicated by arrows labeled with capital letters, while nodes in the gap are indicated by arrows labeled with greek letters. The global maximum of the gap magnitude (labeled A or D) resides in all cases on the elliptic part of the Fermi surface, while the second largest maximum (labeled B or E) is located on the quasi-one-dimensional part. A third smallest maximum (labeled C) is possible on the quasi-1D sheet. In the \( d_{xy} \) case, the nodes labeled \( \gamma \) appear in addition to the expected set of nodes \( \delta \), because the Fermi surface touches the Brillouin zone boundary. The \( \gamma \) nodes lead to the second maximum (labeled E) of the gap magnitude [Figs. 8(a)–8(c), middle panel], but they are otherwise irrelevant for the low-energy physics. As the three possible gap structures share two maxima of slightly different size, the simulated quasiparticle DOS looks quite generic [Figs. 8(a)–8(c), bottom panel]. A two-peak structure is observed far away from the Fermi level, which corresponds to the energy values of the two largest maxima in the gap magnitude.

Important differences are revealed, however, at low energies: the \( d_{xy} \) state is featurelessly V-shaped [Fig. 8(c), bottom panel], while the spectrum of the eight-node state has an additional peak close to 2 meV [Fig. 8(b), bottom panel], which is linked to the small gap (labeled C) on the quasi-1D sheet. This leads to an outer and an inner V-shape with different slopes. For the four-node \( s_\pm + d_{x^2−y^2} \) state we observe a peculiar dip around 1 meV in the quasiparticle spectrum [Fig. 8(a), bottom panel]. This corresponds to the minimum value of the gap magnitude on the quasi-one-dimensional part of the Fermi surface. Inside of this dip, a V-shaped region emerging from the \( \beta \) nodes is again observed.

We emphasize that our predictions are to be taken as qualitative, not quantitative, regarding the overall energy scale and the relative gap sizes. The main features explained above, however, are robust. The detection of such low-energy structures is certainly not an easy task, but we believe it will be possible with state-of-the-art equipment and proper sample preparation.

IV. CONCLUSIONS

In summary, we investigated the superconducting state of \( \kappa \)-(ET)\(_2\)X charge-transfer salts in an individual molecule model based on a combination of \textit{ab initio} density-functional theory and random-phase approximation spin-fluctuation calculations. We obtained kinetic parameters of the molecule Hamiltonian for eight superconducting \( \kappa \)-type materials using
projective Wannier functions. We found that the superconducting order parameter in a realistic molecule model is different from the one in the usual dimer-approximated Hamiltonian for all investigated materials. The superconducting phase diagram of the molecule description is dominated by an extended $s + d_{x^2−y^2}$ symmetry that emerges from the competition between squarelike and diagonal hopping processes on the original $\kappa$-type lattice, while the physics of the dimer model is reproduced also for finite dimerization in the limit of infinite dimerization.

Furthermore, the $s_\pm + d_{x^2−y^2}$ state, which features nodes both along the crystallographic axes and the Brillouin zone diagonals, might explain the multitude of contradictory experimental results regarding the nodal positions. We also simulated tunneling spectroscopy experiments for all nodal configurations encountered in our phase diagram. The difference between those pairing states unfortunately manifests itself only at very low energies, making experimental detection difficult, but not impossible. Based on the \textit{ab initio} calculated model parameters, we found that the well-studied material $\kappa$-(ET)$_2$Cu[N(CN)$_2$]Br is situated near the phase-transition configuration encountered in our phase diagram. The difference between those pairing states unfortunately manifests itself only at very low energies, making experimental detection difficult, but not impossible. Based on the \textit{ab initio} calculated model parameters, we found that the well-studied material $\kappa$-(ET)$_2$Cu[N(CN)$_2$]Br is situated near the phase-transition line between $s_\pm + d_{x^2−y^2}$ and $d_{xy}$ superconducting states, which supports the interpretation of recent scanning tunneling spectroscopy experiments.

A question unanswered by our study is why superconducting transition temperatures among quasi-two-dimensional charge-transfer salts can differ by more than a factor of 4. As there is no obvious connection between $T_c$ and the parameters of the kinetic Hamiltonian, a method that can qualitatively reproduce the ordering of transition temperatures in real materials is required to elucidate this issue.

In conclusion, we believe that a significant part of the physics in quasi-two-dimensional charge-transfer salts has unfortunately been overlooked so far, because theory has

FIG. 8. Gap function on the Fermi surface (top panel), magnitude of the gap function vs angle measured with respect to the $k_x$ direction (middle panel), and simulated quasiparticle density of states in the superconducting state (bottom panel). In all cases, we assumed an energy scale $\Delta_0 = 10$ meV. Only $t_3/t_1$ is varied. Other parameters are fixed to $t_2/t_1 = 0.4375$ and $t_4/t_1 = 0.1$. Maxima of the superconducting gap magnitude are labeled with upper-case letters. Nodes of the superconducting order parameter are labeled with greek letters. All nodes and maxima not labeled explicitly are symmetry-equivalent to the labeled ones. Column (a) shows the case of $s_\pm + d_{x^2−y^2}$ symmetry with four nodes ($t_3/t_1 = 0.5$). Column (b) shows the results for $s_\pm + d_{x^2−y^2}$ symmetry with eight nodes ($t_3/t_1 = 0.3475$). Column (c) shows the $d_{xy}$ case ($t_3/t_1 = 0.25$).
adhered to the dimer model for too long, and too many experiments have been interpreted based on a dichotomy of $d_{x^2-y^2}$ and $d_{3z^2-r^2}$ states, which is inappropriate for the orthorhombic lattice realized in $\kappa$-(ET)$_2$X materials.

It is an interesting open question whether the magnetic, insulating, and possible quantum spin-liquid states known on a dimer/molecule with site index $i$, where states $|i\rangle$ denote the orbitals living on a dimer/molecule with site index $i$. Only unique matrix elements are listed. The rest of the elements are generated by using $\langle i | H_{\text{hop}} | j \rangle = \langle j | H_{\text{hop}} | i \rangle^*$. In all models, there is only one orbital per lattice site.

To obtain the correct electron filling, one has to introduce a chemical potential $\mu$, so that $H_0 = H_{\text{hop}} - \mu \sum_i c_i^\dagger c_i$ is half-filled for the dimer model in either representation and $3/4$-filled for the molecule model.

The single-band representation of the dimer model is given by

$$H_{\text{hop}}(\vec{k}) = 2t[\cos(k_xa) + \cos(k_yb)]$$

$$+ 2t'[\cos(k_xa)\cos(k_yb) - \sin(k_xa)\sin(k_yb)].$$  \hfill (A1)

The two-band representation of the dimer model can be written as

$$\langle 0 | H_{\text{hop}} | 0 \rangle = \langle 1 | H_{\text{hop}} | 1 \rangle = 2t' \cos(k_xa),$$  \hfill (A2a)

$$\langle 0 | H_{\text{hop}} | 1 \rangle = 2t(1 + e^{ikxa} + e^{ikyb} + e^{ikya}e^{ikxb}).$$  \hfill (A2b)

The four-band molecule model is given by

$$\langle 0 | H_{\text{hop}} | 1 \rangle = t_1 + t_3 e^{ikxa},$$  \hfill (A3a)

$$\langle 0 | H_{\text{hop}} | 2 \rangle = t_4(1 + e^{-ikxa}),$$  \hfill (A3b)

$$\langle 1 | H_{\text{hop}} | 3 \rangle = t_5(1 + e^{-ikxa}),$$  \hfill (A3c)

$$\langle 1 | H_{\text{hop}} | 2 \rangle = t_5 e^{-ikya}(1 + e^{-ikxa}),$$  \hfill (A3d)

$$\langle 1 | H_{\text{hop}} | 3 \rangle = t_5 e^{-ikxa}(1 + e^{-ikya}),$$  \hfill (A3e)

$$\langle 2 | H_{\text{hop}} | 3 \rangle = t_1 + t_3 e^{-ikya}.\hfill (A3f)

Here, diagonal entries $\langle i | H_{\text{hop}} | i \rangle$ are zero for all $i$.

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APPENDIX: MATRIX ELEMENTS OF THE KINETIC HAMILTONIANS

For completeness, we list the kinetic part of the dimer model in one- and two-band representation, as well as the kinetic part of the four-band molecule model. We denote the unit-cell parameters in the $x$ and $y$ directions as $a$ and $b$, respectively. The multiband Hamiltonians are given as matrix elements $\langle i | H_{\text{hop}} | j \rangle$, where states $|i\rangle$ denote the orbitals living on a dimer/molecule with site index $i$. Only unique matrix elements are listed. The rest of the elements are generated by using $\langle i | H_{\text{hop}} | j \rangle = \langle j | H_{\text{hop}} | i \rangle^*$. In all models, there is only one orbital per lattice site.

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The single-band representation of the dimer model is given by

$$H_{\text{hop}}(\vec{k}) = 2t[\cos(k_xa) + \cos(k_yb)]$$

$$+ 2t'[\cos(k_xa)\cos(k_yb) - \sin(k_xa)\sin(k_yb)].$$  \hfill (A1)

The two-band representation of the dimer model can be written as

$$\langle 0 | H_{\text{hop}} | 0 \rangle = \langle 1 | H_{\text{hop}} | 1 \rangle = 2t' \cos(k_xa),$$  \hfill (A2a)

$$\langle 0 | H_{\text{hop}} | 1 \rangle = 2t(1 + e^{ikxa} + e^{ikyb} + e^{ikya}e^{ikxb}).$$  \hfill (A2b)

The four-band molecule model is given by

$$\langle 0 | H_{\text{hop}} | 1 \rangle = t_1 + t_3 e^{ikxa},$$  \hfill (A3a)

$$\langle 0 | H_{\text{hop}} | 2 \rangle = t_4(1 + e^{-ikxa}),$$  \hfill (A3b)

$$\langle 1 | H_{\text{hop}} | 3 \rangle = t_5(1 + e^{-ikxa}),$$  \hfill (A3c)

$$\langle 1 | H_{\text{hop}} | 2 \rangle = t_5 e^{-ikya}(1 + e^{-ikxa}),$$  \hfill (A3d)

$$\langle 1 | H_{\text{hop}} | 3 \rangle = t_5 e^{-ikxa}(1 + e^{-ikya}),$$  \hfill (A3e)

$$\langle 2 | H_{\text{hop}} | 3 \rangle = t_1 + t_3 e^{-ikya}.\hfill (A3f)

Here, diagonal entries $\langle i | H_{\text{hop}} | i \rangle$ are zero for all $i$.


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