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Exercise Set 12
(Due date: Tuesday, February 2, 2010)

Exercise 24 (Extrinsic semiconductors) (15 points)

Due to doping with N_d donor atoms, donor levels appear in a seminconductor density of states that can be empty or singly occupied, and the average number of electrons is given by

$$n_d = \frac{N_d}{\frac{1}{2}e^{\beta(E_d-\mu)}+1} \tag{1}$$

where $\beta = (k_B T)^{-1}$.

(a) Show that if the energy of a doubly occupied donor level is taken to be $2E_d+U$, then Eq. (1) must be replaced by

$$n_d = N_d \frac{1+e^{-\beta(E_d-\mu+U)}}{\frac{1}{2}e^{\beta(E_d-\mu)}+1+\frac{1}{2}e^{-\beta(E_d-\mu+U)}} \tag{2}$$

(b) Verify that Eq. (2) reduces to Eq. (1) as $U \to \infty$ which means that double occupancy becomes forbidden.

(c) Consider a donor impurity with many bound electronic orbital levels, with energy E_i. Assuming that the electron-electron Coulomb repulsion prohibits more than a single electron from being bound to the impurity, show that the appropriate generalization of Eq.(1) is

$$\frac{N_d}{1+\frac{1}{2}(\sum e^{-\beta(E_i-\mu)})^{-1}}. \tag{3}$$

Exercise 25 (Slater determinant) (15 points)

Consider Fermi creation operators $c_k^\dagger$ for states with the wave function

$$\langle r|k\rangle = \langle 0|c_r c_k^\dagger|0\rangle \equiv \phi_k(r)\,.$$

Calculate explicitly the Slater determinant $\langle r_1, r_2, r_3 | k_1, k_2, k_3 \rangle$ for three electrons. Here,

$$c_k^\dagger = \sum_{r_i} e^{ikr_i} c_{r_i}^\dagger$$

and

$$\langle r_1, r_2, r_3 | k_1, k_2, k_3 \rangle = \langle 0 | c_{r_1} c_{r_2} c_{r_3} c_{k_1}^\dagger c_{k_2}^\dagger c_{k_3}^\dagger | 0 \rangle \, .$$