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Exercise Set 11

(Due date: Tuesday, January 26, 2010)

Exercise 22 (**Free electron gas in two dimensions**) (15 points)

a) Show that in two dimensions the free electron density of states $\rho(E)$ is a constant independent of E for $E > 0$ and 0 for $E < 0$. What is the constant?

b) Show that because $\rho(E)$ is constant, every term in the Sommerfeld expansion for Z_e except the $T = 0$ term vanishes. Deduce that $\mu = E_F$ at any temperature.

c) Deduce from

$$Z_e = \int_{-\infty}^{\infty} dE\, \rho(E) f(E)$$

that for the constant $\rho(E)$ of a)

$$\mu + k_B T \ln\left(1 + e^{-\frac{\mu}{k_B T}}\right) = E_F$$

How much does μ differ from E_F at low temperature? What does that mean for the Sommerfeld expansion?

Exercise 23 (**Electrons in copper**) (15 points)

Determine for the metal copper the fraction of electrons whose energy at room temperature is larger than $E_F - 2k_B T$.

Hint: The Fermi temperature for copper is

$$T_F = \frac{E_F}{k_B} = 8.1 \cdot 10^4 \text{ K}\,.$$

For $T \ll T_F$ the Fermi function is approximately a Heaviside function

$$f(E, T \ll T_F) = \theta(E - E_F)\,.$$

The derivative of a Heaviside function is a Dirac delta function:

$$f'(E, T \ll T_F) = -\delta(E - E_F)\,.$$