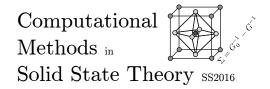
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## Exercise Set 4

(Due date: Monday, June 13, 2016)

## **Exercise 4** (Exact diagonalization of spin models) (10 points)

a) Consider the Heisenberg dimer with spin-1/2.

$$\hat{\mathsf{H}} = \mathsf{J}\,\vec{\mathsf{S}}_1\cdot\vec{\mathsf{S}}_2$$

Use the basis states  $|\uparrow\uparrow\rangle$ ,  $|\uparrow\downarrow\rangle$ ,  $|\downarrow\uparrow\rangle$ ,  $|\downarrow\downarrow\rangle$  to find the eigenvalues and eigenvectors of the Hamiltonian analytically. Which is the ground state of the system for J = 1?

b) Write a program that evaluates a linear spin-1/2 chain with antiferromagnetic next-neighbour Heisenberg interactions and periodic boundary conditions using full diagonalization of the Hamiltonian.

$$\hat{H} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

- c) Implement the Lanczos algorithm for the Heisenberg chain. Verify your results with the program written in b).
- d) Consider the Hamiltonian of the antiferromagnetic next-neighbour Heisenberg chain. Introduce a magnetic field H and set the gyromagnetic ratio to g = 2.

$$\hat{H} = J \sum_{} \vec{S}_i \cdot \vec{S}_j - g \mu_B H \sum_i S_i^z$$

Plot the average magnetization per lattice site as a function of the applied magnetic field for different lengths of the chain. What do you observe?