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Frankfurt, June 19, 2012

Exercises for Computational Methods in Solid State Theory SS 2012  $\,$ 

## Exercise Set 8

(Due date: Monday, June 25, 2012)

## Exercise 8 (Exact diagonalization) (10 points)

The Hamiltonian of single-band Hubbard model with frustration is given as

(1) 
$$H = -t \sum_{\langle i,j \rangle,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} - t' \sum_{[i,j'],\sigma} c^{\dagger}_{i\sigma} c_{j'\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow},$$

where  $c_{i\sigma}$  ( $c_{i\sigma}^{\dagger}$ ) is the annihilation (creation) operator of an electron with spin  $\sigma$  at the i-th site, and U represents the Coulomb repulsion. Consider a 6-site toy model with periodic boundary condition as given in the figure below.



- a) Construct the Hamiltonian with  $\frac{1}{3}$ ,  $\frac{2}{3}$  and half-filling for t = t' = 1 and U/t = 6 using a programming language.
- b) Read the Hamiltonians obtained from a) into the Mathematica and calculate the eigenvectors and eigenvalues in the three cases.