# Exercises for Computational Methods in Solid State Theory 

SS 2012
Exercise Set 8
(Due date: Monday, June 25, 2012)

Exercise 8 (Exact diagonalization) (10 points)
The Hamiltonian of single-band Hubbard model with frustration is given as

$$
\begin{equation*}
\mathrm{H}=-\mathrm{t} \sum_{<i, j>, \sigma} c_{i \sigma}^{\dagger} \mathrm{c}_{j \sigma}-\mathrm{t}^{\prime} \sum_{\left[i, j^{\prime}\right], \sigma} c_{i \sigma}^{\dagger} \mathrm{c}_{j^{\prime} \sigma}+u \sum_{i} n_{i \uparrow} n_{i \downarrow} \tag{1}
\end{equation*}
$$

where $c_{i \sigma}\left(c_{i \sigma}^{\dagger}\right)$ is the annihilation (creation) operator of an electron with spin $\sigma$ at the i-th site, and $U$ represents the Coulomb repulsion. Consider a 6 -site toy model with periodic boundary condition as given in the figure below.

a) Construct the Hamiltonian with $\frac{1}{3}, \frac{2}{3}$ and half-filling for $\mathrm{t}=\mathrm{t}^{\prime}=1$ and $\mathrm{U} / \mathrm{t}=6$ using a programming language.
b) Read the Hamiltonians obtained from a) into the Mathematica and calculate the eigenvectors and eigenvalues in the three cases.

