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Exercises for Advanced Physics 3, 2018 term 4

Exercise Set 5 (Due date: Tuesday, January 29, 2019)

Exercise 5 (Coulomb interaction in second quantization) (10 points)

- (a) Read Chapter 5.1 of the script and explain the idea of second quantization in your own words.
- (b) When rewriting the Coulomb interaction using second quantization, we encounter the term

(1)
$$\sum_{\sigma_1 \sigma_2} a^{\dagger}_{\vec{R}m_1 \sigma_1} a_{\vec{R}m_1 \sigma_2} a^{\dagger}_{\vec{R}m_2 \sigma_2} a_{\vec{R}m_2 \sigma_1} a_{\vec{R}m_2 \sigma_2} a_{\vec{R}m_2 \sigma_1} a_{\vec{R}m_2 \sigma_2} a_{\vec{R}m_2 \sigma_1} a_{\vec{R}m_2 \sigma_2} a_{\vec{R}m_2 \sigma_1} a_{\vec{R}m_2 \sigma_2} a_{\vec$$

Use the anticommutation relations of the Fermionic creation (annihilation) operators $a_{\vec{R}m_1\sigma_1}^{\dagger}$ $(a_{\vec{R}m_1\sigma_1})$ and the definitions

$$n_{\vec{R}m} = \sum_{\sigma} a_{\vec{R}m\sigma}^{\dagger} a_{\vec{R}m\sigma}$$

$$(2) \qquad s_{\vec{R}m}^{\alpha} = \sum_{\sigma\sigma'} a_{\vec{R}m\sigma}^{\dagger} \frac{\sigma_{\sigma\sigma'}^{\alpha}}{2} a_{\vec{R}m\sigma'} \quad (\text{Pauli matrices } \sigma_{\sigma\sigma'}^{\alpha}, \alpha = x, y, z)$$

$$\vec{s}_{\vec{R}m_2} = \left(s_{\vec{R}m}^{\chi}, s_{\vec{R}m}^{y}, s_{\vec{R}m}^{z}\right)$$

of the number operators $n_{\overrightarrow{R}\mathfrak{m}}$ and the spin operators $s_{\overrightarrow{R}\mathfrak{m}}^{\alpha}$ to show that

(3)
$$\sum_{\sigma_1\sigma_2} a^{\dagger}_{\vec{R}\mathfrak{m}_1\sigma_1} a_{\vec{R}\mathfrak{m}_1\sigma_2} a^{\dagger}_{\vec{R}\mathfrak{m}_2\sigma_2} a_{\vec{R}\mathfrak{m}_2\sigma_1} = \frac{1}{2} n_{\vec{R}\mathfrak{m}_1} n_{\vec{R}\mathfrak{m}_2} + 2\vec{s}_{\vec{R}\mathfrak{m}_1} \cdot \vec{s}_{\vec{R}\mathfrak{m}_2}$$